Implementation of the optimal mechanism through name-your-own-price selling

Robert Zeithammer

November 6, 2013

Abstract: This paper models a name-your-own-price (NYOP) seller who commits to a schedule of bid-acceptance probabilities before his marginal cost of production is realized. After a buyer submits a bid, the seller learns his marginal cost and decides whether to accept the bid or reject it. This paper shows constructively that under standard regularity conditions, such an NYOP seller can implement the first-best ex-post optimal mechanism: for every cost realization, the seller makes as much profit as he would if he could learn his cost first and use the optimal mechanism contingent on it. This ex-post optimality is robust to an arbitrary competing price in the outside posted-price spot market for the same good, and to opacity of the NYOP offering. We characterize both the optimal bid-acceptance probability and its associated bidding function. The bidding function involves a jump discontinuity driven by the competition with the outside market, and it can be locally decreasing in valuation when the NYOP offering is opaque. Implementability of the optimal mechanism depends on seller commitment to reject relatively low bids above cost, and accept relatively high bids below cost. In an extension, we describe and compare two alternative strategies available to sellers with limited commitment: commitment to a minimum bid and commitment to a participation fee.

Contact: Robert Zeithammer, UCLA Anderson School of Management. rzeitham@ucla.edu

1. Introduction

It is well known that a monopolist selling one indivisible object to a single risk-neutral buyer should set a posted price and make a take-it-or-leave-it offer. Specifically, Riley and Zeckhauser (1983) show that such a posted-price strategy is "optimal in comparison to any other, including all forms of buyer involvement such as quoting offers" (p. 289). In light of this classic result, the emergence of name-your-own-price (NYOP) sellers, such as Priceline.com, requires theoretical justification because it involves buyers quoting offers and the seller accepting or rejecting them based on his own private cost information. Moreover, the literature to date contains only limited results about how an NYOP seller can maximize profits (e.g. Fay 2004; Spann, Zeithammer, and Haübl 2010; Shapiro 2011), and the optimal NYOP selling strategy has yet to be characterized. In this paper, we derive the optimal strategy, outline its implementation, and provide a theoretical justification for NYOP selling.

One theoretical justification for NYOP selling is as a second-best solution to the optimal selling problem that accommodates buyer activism enabled by the Internet: when an active buyer makes the first move in her interaction with a seller by submitting a bid, the seller does not know his marginal cost until after receiving the bid (e.g., as in the business model of Priceline.com that solicits supplier quotes only after receiving a buyer bid; see Anderson 2009). The seller would wish to learn his cost first and post the appropriate take-it-or-leave-it offer in advance of the interaction (Riley and Zeckhauser 1983), but buyer activism (perhaps along with a preference for bidding instead of considering posted prices) prevents him from doing so. Instead, the NYOP seller can only declare a schedule of bid-acceptance probabilities for different bid levels, but not as a function of his (yet unknown) production cost.

We show that under standard regularity assumptions about the distribution of buyer valuations, NYOP selling can accommodate the aforementioned buyer activism (along with its associated cost uncertainty) without compromising seller profits. Despite having to set his selling strategy (i.e., his schedule of bid-acceptance probabilities) before learning his cost of production, the NYOP seller can achieve first-best ex-post profits. In other words, for every possible cost realization, the NYOP seller can make as much profit as he would if he could learn his marginal cost upfront, and use the optimal contingent mechanism (posted pricing with a price contingent on the cost realization). The buyer does not extract any additional expected surplus from the seller's ex-ante uncertainty. The ex-post revenue equivalence arises because NYOP selling can implement the first-best abstract optimal mechanism of Riley and Zeckhauser (1983). Consequently, NYOP does not have to be justified as some sort of second-best way of selling when standard posted pricing is not feasible. We now briefly outline our model before giving intuition for the main result and introducing several additional findings regarding the optimal NYOP selling strategy.

Following Spann, Zeithammer, and Haübl (2010) and Shapiro (2011), our model of the market in which the NYOP seller operates assumes the object sold by the NYOP seller can also be obtained in an outside spot market for some commonly known price. For example, the object sold can be a seat on a particular flight[1] sold by Priceline (the NYOP seller), and the outside market can consist of the lowest price for that seat posted by travel retailers (e.g. Expedia, Travelocity, Orbitz, etc.). This outside price, along with the distribution of costs below it, is the only information the seller has when he sets his strategy. Unlike in standard pricing, he only learns his marginal cost of production after receiving the buyer's bid.

To derive the optimal NYOP strategy in our setting, we adapt mechanism-design techniques to accommodate the seller's ex-ante lack of cost information. We first derive the optimal direct revelation mechanism and then characterize its NYOP implementation. In the direct-revelation mechanism, the optimal allocation rule *conditional* on a cost realization is not surprising: the seller should accept buyer valuations above the monopoly price implied by the realized cost. However, NYOP selling is not a direct-revelation mechanism, because successful NYOP buyers pay their bids and hence bid strictly below their valuations. Interestingly, we find that the optimal allocation rule from the direct-revelation mechanism can be implemented even

[1] In addition to allowing buyer bids as in NYOP, Priceline also makes its offering opaque (Fay 2008) by hiding the airline name and exact time of departure. We analyze both a non-opaque NYOP offering (in the main model) and an opaque offering (in an extension).

when buyers pay their bids and bid strategically. Hence follows our main result described above: NYOP can be ex-post optimal for the seller. In other words, we show a pair of functions {bidding function, bid-acceptance probability function} exists that best respond to each other and implement the optimal mechanism ex post.

The key intuition behind the ex-post optimality of NYOP is that although the seller does not know his cost at the time of setting his strategy, he learns it before making the bid-acceptance decision, and hence he can compare it to the buyer's valuation.[2] How does the seller know the buyer's valuation? We show that when the distribution of valuations in the population of buyers is regular in the sense of Myerson (1981), an invertible bidding function exists that best responds to the optimal NYOP strategy. At the time of the bid-acceptance decision, the seller thus has all the pieces of information needed for implementation of the optimal allocation. The outside spot market influences both the buyer's bidding strategy and the seller's bid-acceptance probability, but it does not hinder implementation.

Our second contribution is a characterization of the pair of functions that implement the optimal mechanism within NYOP selling. The optimal bid-acceptance function involves a minimum bid in that it rejects all bids under the monopoly price corresponding to the lowest possible cost. Above the minimum level, higher bids are accepted with higher probability up to the bid level of the buyer just under the outside market posted price, who faces a probability of acceptance strictly below one. All "high-valuation" buyers with valuations above the outside market price mimic the buyer with valuation exactly equal to it, and the optimal allocation rule calls for the seller to accept their bid with certainty. To ensure buyers just below the outside market price do not deviate to mimic the high-valuation buyers, the bidding function must involve a jump discontinuity, and the probability acceptance rule must rise up to unity sufficiently slowly. As a consequence, optimal NYOP selling involves a range of intermediate bids that nobody submits, and a bid strictly below the outside market price that is submitted by a

[2] Note, however, that the optimal NYOP strategy does not require the seller to credibly communicate his cost realization or commit to any action contingent on a cost realization. Instead, it relies on commitment to a probability-acceptance function that depends on the bid level alone.

positive measure of buyers and accepted with certainty.

In our first extension, we extend our main revenue-equivalence and implementation result to an NYOP seller who makes his offering opaque (Fay 2008) and continues to face an outside market in which the object is sold without opacity. Specifically, we show that an NYOP seller of an opaque offering can make the same profit as a posted-price seller of the same opaque offering who knows his cost in the beginning of the game. The bidding strategy that implements the optimal mechanism is striking: not only does it also involve a jump discontinuity, but it is also decreasing on a range of valuations. Unlike in the baseline case without opacity, the "high-valuation" buyers with valuations above the outside market price do not pool together, and their *net* valuations of the NYOP offering (and hence their bids) are decreasing in their valuations of the non-opaque outside market offering.

Just like a classic monopolist needs to commit to rejecting profitable trades with buyers below the monopoly price, the optimal NYOP seller needs to commit to sometimes rejecting bids above his cost. Surprisingly, the optimal NYOP seller also needs to commit to sometimes selling below cost, because the bid high-valuation buyers use is accepted with certainty and is strictly below the highest possible cost. One way the seller may create such commitment to a probabilistic assignment would be to post a table with the probabilities of different bids being accepted, and allow buyers to report bids and acceptance outcomes to a third-party auditor. The auditor could then post the summary statistics, ensuring the empirical probabilities match the promised ones.

However, the above amount of commitment may be too strong for some real-world markets. In the final section of the paper, we characterize and compare two strategies available to "limited commitment" sellers: the minimum-bid strategy akin to a public reserve in an auction, and the participation fee strategy—a two-part-tariff strategy proposed by Spann, Zeithammer, and Haübl (2010). We find that the relative profitability of the two limited-commitment strategies depends on the level of the outside price: when the outside price is high, the participation fee dominates the minimum bid, and vice versa. We also show that the relative advantage of full commitment over minimum-bid-only vanishes when the outside price is low.

2. Related literature

NYOP selling has generated a lot of academic interest as a novel environment for studying consumer decision-making (e.g. Chernev 2003, Ding et al. 2005, Spann and Tellis 2006). This paper contributes to the small but growing theoretical literature about optimal NYOP selling. The most closely related paper is by Spann, Zeithammer, and Haübl (2010), who show that an NYOP seller analogous to the one assumed herein profits more from charging a participation fee than from charging a minimum markup or from some combination of a fee and a minimum markup. Whereas they assume both distributions that parameterize the model are uniform, we prove our results in full generality. Compared to their seller, the present seller needs more commitment and receives a strictly higher profit. Specifically, we show that no participation fee can implement the same allocation as the optimal direct-revelation mechanism, and so employing participation fees is less profitable than employing our optimal mechanism. Because participation fees require less commitment, we consider them when we discuss limited-commitment sellers in our second extension.

In another closely related work, Shapiro (2011) starts from the same Riley and Zeckhauser (1983) quote as this paper, and shows that risk aversion is another way to rationalize the existence of NYOP selling. Specifically, he shows that the NYOP monopoly profit is higher than the posted-price monopoly profit when buyers are risk averse, because such buyers bid more than risk-neutral buyers to avoid the risk of not winning at all. Shapiro's (2011) model makes several of the same assumptions we do: his seller can commit to a probabilistic bid acceptance, and he sometimes faces a non-strategic premium posted-price seller akin to the one we assume. In contrast to Shapiro (2011), our buyers are risk neutral and our seller is ex-ante uncertain about his wholesale cost. Extending our model with ex-ante seller uncertainty to the case of risk-averse buyers is not tractable to us, but Shapiro's logic suggests such an NYOP seller would strictly outperform his posted-price counterpart instead of merely getting the same profit as he does under risk neutrality.

Several other papers rationalize NYOP as a best response in a competitive setting (Fay 2009) or as a price-discrimination tool (Wang, Gal-Or, and Chatterjee 2009; Shapiro and Zillante 2009). Such models rely on buyer heterogeneity in an inherent preference for buying via NYOP, perhaps because of varying frictional costs (Hann and Terwiesch 2003) or the varying impact of the risk arising from opacity of the NYOP offering (Fay 2009). The buyers in this paper instead care only about their surplus, and do not favor posted-price buying over NYOP, or vice versa. We do address opacity in our first extension, where all buyers strictly prefer the offering of the outside market because it is not opaque.

Chernev (2003) and Spann et al. (2012) show that consumer behavior is different when the NYOP seller allows any price to be named (a true "name your own price" in Chernev's nomenclature) compared to when the NYOP seller presents a menu of prices (called "select your price" by Chernev). The mechanism proposed here is more in line with "select your price." Moreover, we suggest the seller should present not only a menu of prices, but also the acceptance probabilities. Such an institution facilitates commitment, simplifies bidding, and enables the seller to better learn consumer preferences.

NYOP selling is also related to auction theory in that it corresponds to a first-price sealed-bid auction with a single bidder (the buyer) and a stochastic secret reserve (the seller's bid-acceptance function). The implementability of the optimal mechanism via NYOP selling can thus be interpreted as an extension of the revenue equivalence between first-price and second-price auctions, itself a well-known result in auction theory (Vickrey 1961, Myerson 1981): the posted-price mechanism is equivalent to a set of second-price sealed-bid auctions, each with an optimal reserve price and only one bidder. Optimal NYOP selling can be thought of as a set of first-price sealed-bid auctions, each of which is constructed to accept a bid exactly often enough to maintain revenue equivalence with the corresponding second-price auction.

We analyze a static model in which each buyer is restricted to a single bid, but real-world bidders may manage to bid multiple times. Fay (2004) uses a stylized model to show that repeat bidding does not necessarily erode seller profits, as long as the seller is aware of the behavior and uses the right dynamic thresholds. Extending our setup to a dynamic environment is beyond the scope of this paper, but we propose that something akin to Fay's (2004) finding would replicate: as long as the seller anticipates repeat bidding and conditions his acceptance-probability strategy on the number of bids the bidder has submitted to date, the seller should be able to extract first-best profits from the repeated interactions.

3. Model

Our notation follows Krishna (2002) whenever possible, and it is summarized for easy reference in Table 1. A risk-neutral buyer is interested in buying one particular indivisible object. The buyer's valuation x of the object is drawn from a continuous distribution $F(x)$ with density $f(x)$ and support on $[\underline{x}, \overline{x}]$. Assume the virtual value $\psi(x) \equiv x - \frac{1-F(x)}{f(x)}$ is increasing, that is, that the distribution F is *regular* in the sense of Myerson (1981).

The object is readily available in an outside posted-price market for a commonly known price $\psi^{-1}(0) < p \leq \overline{x}$, where $\psi^{-1}(0)$ is the price a monopolist with zero marginal cost would charge for the object. When the buyer does not buy the object from either seller, her payoff is zero. Following Spann, Zeithammer, and Haübl (2010) and Shapiro (2011), we assume the NYOP seller is small in that he takes the posted price as fixed, and the outside spot market does not adjust its posted price in response to the NYOP seller's strategy.

An NYOP seller can procure the object for a wholesale cost $c \sim H(c)$, where the distribution H has full support on $[0, p]$. In other words, the posted price is a public upper bound on the NYOP seller's wholesale cost. For example, the NYOP seller can be selling excess capacity on a particular flight, whereas p is the price of a seat on the same flight posted by Expedia.[3] The entire model is thus parameterized by the two distributions F and H, where the

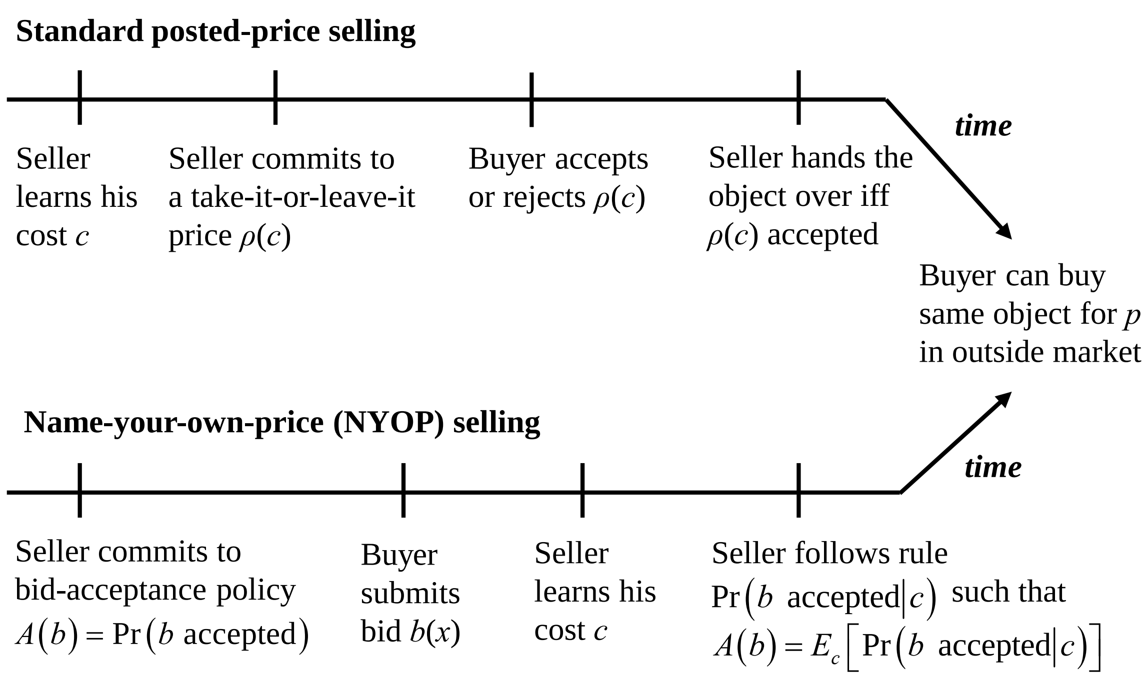
[3] The underlying assumption is that the NYOP seller does not have a special technology for producing the object, but rather obtains the object from the same supplier as his posted-price competitors. Even after learning the posted price, uncertainty about c remains because p is a relatively stable price, set to reflect long-run revenue-management considerations and quite possibly a larger set of customers.

support of the latter depends on p. A lower p is both bad news (tougher competition) and good news (lower expected cost) for the NYOP seller.

Note that in our baseline model, we abstract from the fact that NYOP products are often opaque—a feature pioneered by Priceline, and a potential source of differentiation between the two sellers (Fay 2008, Shapiro and Shi 2008). In our seat-on-a-flight example, the buyer knows the same amount of information about the seat before purchasing it from either seller. In an extension, we show that opacity does not change our main result qualitatively.

Timing of the game is as follows (please see the bottom timeline in Figure 1): In the beginning of the game, the NYOP seller then announces his bid-acceptance strategy $A(b) \equiv \Pr(accept\ b)$ for all possible levels of bid b submitted by the buyer for the object. The buyer then submits a binding bid. After receiving a bid, the seller queries his suppliers for a cost quote to learn his actual c and decides whether to accept the bid. At any time during the game, the buyers can choose to buy from the posted-price outside market and pay the price p. Figure 1 highlights the key contrast with posted pricing in the timing of the cost information.

Figure 1: Timing of the NYOP game, as compared to standard posted pricing



To derive the optimal mechanism, we follow Myerson (1981) and the rest of the mechanism-design literature, and assume the NYOP seller can commit to any bid-acceptance strategy. In our second extension, we relax this assumption in several ways and explore the impact on seller profits. When the seller has no commitment ability, he obviously accepts all bids above *c*, so everyone knows $A(b) = \Pr(b > c)$, and the seller's announcement in the beginning of the game contains no new information for the buyers.

Before solving for the optimal bid-acceptance strategy, we summarize the impact of the outside spot market on the demand the NYOP seller faces. For all bid-acceptance strategies, buyers with $x>p$ all mimic the type $x=p$, because they have a real option to buy in the outside market when the price they name is rejected. In other words, the NYOP seller faces buyers with a distribution of *net* valuations F on $[0, p)$ and $[1 - F(p)]$ mass at p. To see this fact, note the expected surplus U of an $x>p$ buyer who bids b is

$$U(b,x) = A(b)(x-b) + [1 - A(b)](x-p) = (x-p) + U(b,p) \qquad (1)$$

where $U(b,p)$ is the utility of the buyer with $x=p$. It is immediate that the same b that maximizes $U(b,p)$ also maximizes $U(b,x)$. Intuitively, the buyer thus receives all of his valuation in excess of p as surplus, and his participation with the NYOP seller is akin to free gambling in hopes of randomly getting a price below p.

4. Optimal direct-revelation mechanism

We use the revelation principle (Myerson 1981) to restrict attention to direct-revelation mechanisms whereby the buyer reports her valuation truthfully. Much of the material in this section is standard, so the details are relegated to the Appendix. The only deviation from a textbook treatment (e.g., Krishna 2002) is the fact that the seller does not know his cost *c* when he sets his strategy, but he does learn *c* before making his bid-acceptance decision. Knowing the cost before the acceptance decision needs to be made allows us to first optimize the contingent bid-acceptance rule $\pi(x,c) = \Pr(\text{allocate object to } x \text{ when cost is } c)$. Not knowing the cost at the outset restricts the seller to probabilistic assignments that are only a function of *x*. Let $q(x)$ be the probability that a buyer with *x* receives the object. A seller who is planning to use a given $\pi(x,c)$ can only commit to $q(x)=\int_0^p \pi(x,c)\,dH(c)$. The aforementioned atom at *p* also provides an interesting wrinkle relative to the textbook in that the optimal *q* is discontinuous at *p*. The optimal contingent bid-acceptance rule is as follows (please see the Appendix for all proofs):

Proposition 1: *The optimal contingent bid-acceptance rule in a direct-revelation mechanism is*

$$\pi(x,c)=\begin{cases}1 \; when\left(x<p \; and \; c<\psi(x)\right) \; or \; when \; x=p \\ 0 \; otherwise\end{cases}$$

For every cost c, this rule achieves the same profit as a posted-price monopolist who knows c before setting his price, namely, $\int_{\min\left[\psi^{-1}(c),p\right]}^{\bar{x}} \left(\psi(x)-c\right)dF(x)$.

The allocation rule for *x<p* is familiar from the mechanism-design literature: Myerson's (1981) optimal reserve price applies for low buyers: for every *x<p*, the seller should sell (i.e., set $\pi(x,c)=1$) iff $\psi(x)>c \Leftrightarrow x>\text{monopoly price}(c)$. In addition to sometimes serving some of the low buyers, the seller should also always sell to all high-value buyers (*x≥p*) who all bid *p*, because *p≥c* holds for all *c* by construction.

The intuition for $\pi(x,c)$ goes back to a simple posted-price monopolist with a marginal cost *c<p*, who faces demand $\left[1-F(z)\right]$ for all prices *z<p,* and a point mass of $\left[1-F(p)\right]$ customers willing to pay exactly *p.* Such a monopolist charges precisely $\min\left(\psi^{-1}(c),p\right)$ to maximize his profit. Because the seller with commitment can condition his $\pi(x,c)$ on *c*, he can effectively get the *ex-post* monopoly profits. In other words, for every cost *c*, he can get the same profit as a posted-price monopolist who knows *c* before setting his price. Note that the optimal allocation applies regardless of the distribution of seller cost *c*, but the implied bid-acceptance strategy depends on *H.*

Example: F and H Uniform

Throughout the paper, we will use the example of *F*=Uniform[0,1] and *H*=Uniform[0,*p*] to illustrate the findings in closed form. When *F* is uniform on [0,1], $\psi(x)=2x-1$, so the complete optimal allocation is $sell \Leftrightarrow x>\frac{1+c}{2}$ or $x=p$. Note that only buyers with $x>\frac{1}{2}$ have any chance of winning, so the seller effectively sets a minimum bid of ½. The seller makes a profit of $\Pi^*(p)=E_c\left[\int_{\min\left[\frac{1+c}{2},p\right]}^{1}(2x-1-c)\,dx\right]$. When *H* is also uniform, the seller's expected profit becomes $\Pi^*(p)=\int_0^{2p-1}\left(\int_{\frac{1+c}{2}}^{1}\frac{2x-1-c}{p}dx\right)dc+\int_{2p-1}^{p}\left(\int_{p}^{1}\frac{2x-1-c}{p}dx\right)dc=\frac{2p^3+6p(1-p)-1}{12p}$

Note that regularity of *F* in the sense of Myerson (1981) is not required for posted pricing to be the optimal strategy contingent on *c*: Riley and Zeckhauser (1983) show that the optimal $\pi(x,c)$ is a step function with a single step even when $\psi(x)$ is not increasing in *x*. Had we not assumed regularity in this section, Proposition 1 would be modified to $\pi(x,c)=1$ when either $x>x^*$ for some *x** that satisfies $c=\psi(x^*)$, or when *x*=*p* (see Proposition 1 of Riley and Zeckhauser 1983). Exposition is easier with a regular *F*, and the next section will show that regularity is actually a necessary condition for an implementation of the optimal allocation rule through NYOP selling.

The form of the optimal allocation rule in Proposition 1 is not surprising. The key question of this paper is how to implement it within the NYOP institution. The seller could simply promise to charge a price of $\min\left(\psi^{-1}(c),p\right)$ to all bidders with bids that exceed it, preserving the incentive to bid truthfully. In other words, the seller could run a Becker, DeGroot, and Marschak (1964) procedure with a carefully selected distribution of prices. However, an NYOP seller promises to charge buyers their bids whenever a sale occurs. In response to paying their bids, buyers shade their bids below their private valuations. In the next section, we derive the buyer's bidding function and the seller's bid-acceptance rule that implements the optimal mechanism.

5. Implementation of the optimal mechanism through NYOP

An NYOP seller promises to charge buyers their bids whenever a sale occurs. One advantage of accepted buyers paying their bids (vs. the optimal monopoly price conditional on the realized *c*) is that the seller does not need to credibly communicate his *c* to the buyers. However, how buyers will respond is not a priori clear. Will they bid according to an increasing (and hence invertible) bidding function that allows the seller to implement his optimal allocation, or will they somehow obfuscate their type to avoid being exploited? In this section, we show constructively that the buyers' best-response bidding function is invertible—and the NYOP seller can thus implement the optimal allocation rule described in Proposition 1—if and only if the virtual value $\psi(x)$ is increasing in *x*, irrespective of *H*. In other words, regularity of *F* in the sense of Myerson (1981) is necessary and sufficient for an NYOP implementation of the optimal mechanism. The proof proceeds in three steps, each captured in a separate lemma:

1) Derivation of the bidding strategy for low-valuation bidders ($x<p$)
2) Derivation of the bidding strategy for high-valuation bidders ($x=p$)
3) Specification of the optimal bid-acceptance probability for intermediate bid levels

5.1 Bidding strategy for low-valuation bidders (*x*<*p*)

First, consider a buyer with $\psi^{-1}(0)<x<p$ who follows a bidding strategy $\beta(x)$. Proposition 1 shows that his probability of winning in the optimal mechanism is

$q(x)=\int_0^p \pi(x,c)\,dH(c)=H(\psi(x))$. The proof of Proposition 1 shows that his expected utility is

$U(x)=\int_{\underline{x}}^{x} q(t)\,dt=\int_{\underline{x}}^{x} H(\psi(t))\,dt=\int_{\underline{x}}^{x}(x-t)\,dH(\psi(t))$, where the last equality follows from integration by parts. For NYOP selling to be revenue equivalent with the optimal direct-revelation mechanism, the bidding strategy $\beta(x)$ must satisfy:

$$U(x) = q(x)\big(x - \beta(x)\big) \Rightarrow \beta(x) = x - \frac{U(x)}{q(x)} \qquad (2)$$

The structure of the candidate bidding function in equation (2) yields the following result:

Lemma 1: *For bidders with x<p, the NYOP bidding strategy implied by the optimal direct-revelation mechanism of Proposition* 1 *is increasing iff ψ is increasing, and it can be characterized by* $\beta(x) = E_c\left[\psi^{-1}(c) \middle| \psi^{-1}(c) < x\right]$.

Together, Proposition 1 and Lemma 1 show that when *F* is regular, the optimal mechanism can be implemented via NYOP. No additional assumptions about *F* are needed, and the *H* distribution of costs merely needs to have full support on $\left[0, \psi(p)\right]$. The exact $\beta(x)$ function in Lemma 1 follows from plugging the *q* and *U* from Proposition 1 into equation (2).

In words, Lemma 1 shows that the optimal bid by a buyer with valuation *x<p* is the average (over *c*) of monopoly prices the buyer would be willing to pay, namely, prices below *x.* Given the regularity of *F* we assume throughout, the bidding function is increasing, and hence invertible, so the seller can infer each bidder's *x* and apply the allocation rule of Proposition 1. Given the bidding function, we can also solve for the cost-contingent bid-acceptance rule implied by the $\pi(x,c)$ in Proposition 1:

$$c < \psi\left(\beta^{-1}(b)\right) \Leftrightarrow b > \beta\left(\psi^{-1}(c)\right) \Leftrightarrow b > E_{\text{cost}}\left[\psi^{-1}(\text{cost}) \middle| \text{cost} < c\right] \qquad (3)$$

That is, the seller accepts bids over the average monopoly price he would charge for all costs below his actual cost realization.

5.2 Bidding strategy for high-valuation bidders (*x*=*p*)

Now consider a buyer with *x*=*p*, and recall that the optimal allocation rule is to sell to this buyer for all levels of *c*. Because $\psi(x) < x$ for all $x < 1$, the limit of the above acceptance rule as *x* approaches p from below involves an acceptance probability below one for all x other than $x=\underline{x}$ (special because $\psi(\underline{x})=\underline{x}$). In other words, $A(\beta_p^-)=\Pr(c<\psi(p))<1$, where $\beta_p^- = \lim_{x\to p-}\beta(x)$.

To implement the needed $\pi(p,c)=1$, a bid level $\beta(p)>\beta_p^-$ must exist such that buyers with $x=p$ prefer to bid $\beta(p)$, but buyers with $x<p$ do not deviate from bidding according to the above $\beta(x)$. Our next lemma shows that a unique such $\beta(p)$ does exist:

Lemma 2: *For bidders with a net valuation of the NYOP offering equal to the outside price p, the NYOP bidding strategy implied by the optimal direct-revelation mechanism of Proposition 1 is* $\beta(p)=p-\int_{\psi^{-1}(0)}^{p} H(\psi(z))dz>\beta_p^-$.

5.3 Optimal bid-acceptance probability for intermediate bid levels

The optimal mechanism determines the acceptance probability of all bids in $[0,\beta_p^-]$ and $[\beta(p),p]$. No bidders should submit bids in the "intermediate" region of $[\beta_p^-,\beta(p)]$, but the NYOP seller still needs to specify a bid-acceptance strategy function for such bids, ensuring they indeed remain off equilibrium. One simple rule is to simply reject such bids. Another simple rule that keeps $A(b)$ non-decreasing is to simply let $\Pr(accept\ b)=H(\psi(p))$ for all $b\in[\beta_p^-,\beta(p))$. Our next Lemma derives the upper bound on probability acceptance of every intermediate b:

Lemma 3: *When* $A(b)\le\left(\frac{1}{p-b}\right)\int_{\psi^{-1}(0)}^{p} H(\psi(z))dz$ *for* $b\in[\beta_p^-,\beta(p))$, *no buyer submits a bid* $b\in[\beta_p^-,\beta(p))$.

Plugging $b=\beta(p)$ into the bound in Lemma 3 shows that the upper bound approaches 1 as b approaches $\beta(p)$. Therefore, the $A(b)$ can actually be continuous and increasing. Lemmas 1-3 complete the proof of the main result of this paper:

Proposition 2: *For every regular continuous F on* $\left[\underline{x},\bar{x}\right]$ *and every outside posted price* $p>\psi^{-1}(0)$, *the following bid-acceptance probability function implements the ex-post optimal mechanism:*

$$A(b)=\begin{cases} b<\lim_{x\to p-}\beta(x): H\left(\psi\left(\beta^{-1}(b)\right)\right) \\ \lim_{x\to p-}\beta(x)\le b<\beta(p): anything \le \int_{\psi^{-1}(0)}^{p}\frac{H(\psi(z))}{p-b}dz \\ b\ge\beta(p): 1 \end{cases}$$

where the bidding function

$$\beta(x)=\begin{cases} x\in\left[\psi^{-1}(0),p\right): \int_{0}^{\psi(x)}\psi^{-1}(c)\frac{dH(c)}{H(\psi(x))} \\ x=p: p-\int_{\psi^{-1}(0)}^{p}H(\psi(z))dz \end{cases}$$

is the unique best response to $A(b)$. *After learning his production cost c, the seller accepts a bid b whenever* $b>\beta\left(\psi^{-1}(c)\right)$ *for* $b<\lim_{x\to p-}\beta(x)$, *and he accepts* $\beta(p)$ *with certainty.*

Note that the NYOP implementation of the optimal mechanism consists of a pair of a bid-acceptance probability $A(b)$ and a bidding function $\beta(x)$ that best respond to each other. The minimum bid implied by $\beta(x)$ and $A(b)$ is $\psi^{-1}(0)$. It is useful to consider a closed-form example:

5.4 Example: F and H Uniform

It is easy to show that an *F=Uniform*[0,1] implies $\psi^{-1}(0)=\frac{1}{2}$, and hence for $\frac{1}{2}<x<p$,

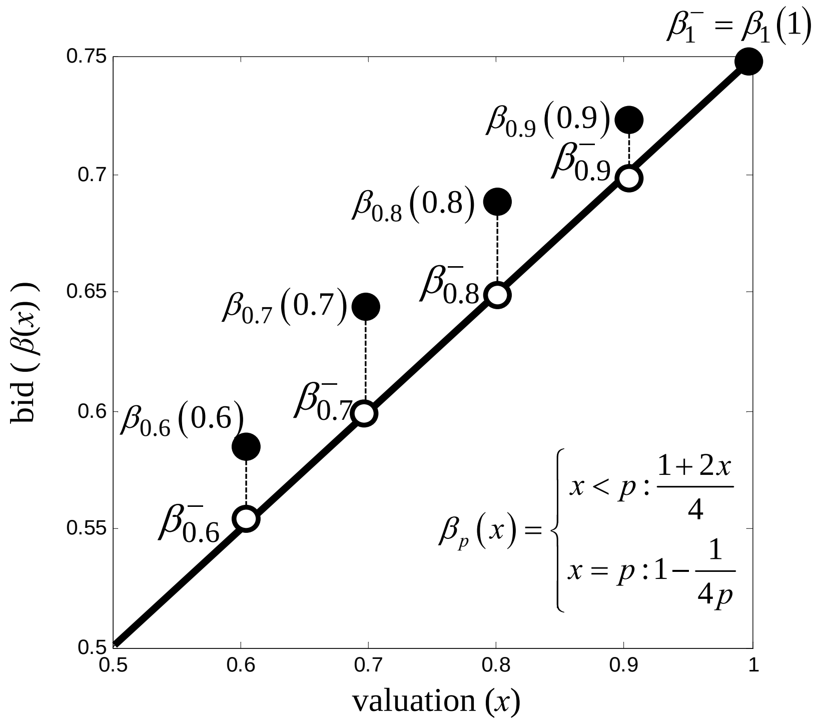
$$\beta(x)=\int_{1/2}^{x}2z\frac{h(2z-1)}{H(2x-1)}dz=\int_{0}^{2x-1}\left(\frac{c+1}{2}\right)\frac{h(c)}{H(2x-1)}dc=E\left(\frac{c+1}{2}\middle|x>\frac{c+1}{2}\right) \quad (4)$$

The bid of the *x=p* bidders is

$$\beta(p)=p\left[1-\Pr\left(p>\frac{c+1}{2}\right)\right]+\Pr\left(p>\frac{c+1}{2}\right)E_c\left(\frac{c+1}{2}\middle|p>\frac{c+1}{2}\right) \quad (5)$$

When H is also uniform on [0,p], $\beta(x)=\frac{1+2x}{4}$ for all $x<p$, and $\beta(p)=1-\frac{1}{4p}$. Please see Figure 2a for the bidding function.

Figure 2a: Bidding functions that implement the optimal mechanism, for five levels of the outside market price *p* $\left(F=Uniform[0,1], H=Uniform[0,p]\right)$



Note to Figure: The subscript of the bidding function indicates p. The empty circles indicate the jump discontinuities from β_p^- to $\beta_p(p)$. For $q<p$, $\beta_q(z)=\beta_p(z)$ for all $z<q$.

Given this bidding strategy, we can also solve for the bid-acceptance rule in closed form:

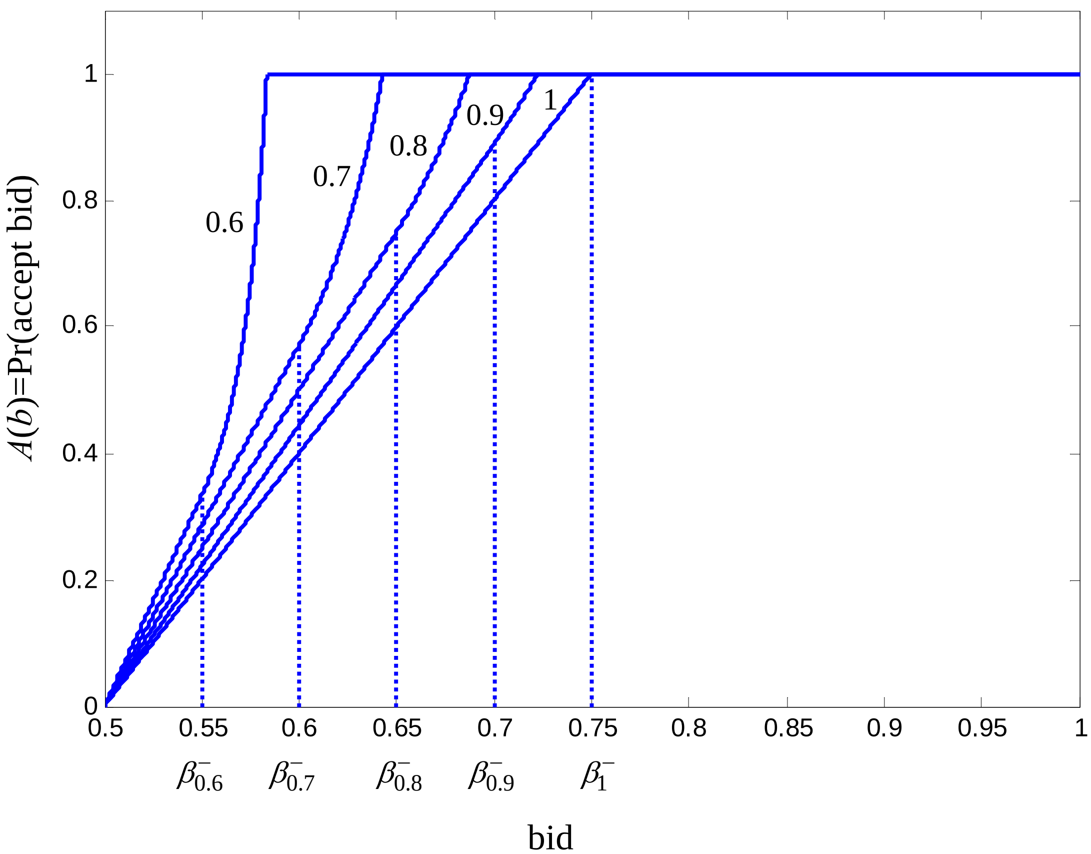
$$\pi(b,c)=1 \Leftrightarrow \begin{cases} b \le \frac{1+2p}{4}: \text{ accept when } c<2\beta^{-1}(b) \Leftrightarrow b>\frac{1}{2}+\frac{c}{4} \\ \frac{1+2p}{4}<b<1-\frac{1}{4p}: \text{accept with Pr} \le \frac{(2p-1)^2}{4p(p-b)} \\ b \ge 1-\frac{1}{4p}: \text{accept always} \end{cases} \quad (6)$$

Therefore, the seller only accepts bids over 1/2, accepts bids over $1-\frac{1}{4p}$ with certainty, and is willing to sell below cost whenever $\frac{1}{2}+\frac{c}{4}<c \Leftrightarrow c>\frac{2}{3}$. The implied bid-acceptance strategy is

$$A(b)=\begin{cases} 0 \text{ for } b<\frac{1}{2} \\ \frac{4b-2}{p} \text{ for } \frac{1}{2}\leq b\leq\frac{1+2p}{4} \\ \text{below } \frac{(2p-1)^2}{4p(p-b)} \text{ for } \frac{1+2p}{4}<b<1-\frac{1}{4p} \\ 1 \text{ for } b\geq 1-\frac{1}{4p} \end{cases} \tag{7}$$

Figure 2b illustrates the optimal A for different levels of p.

Figure 2b: Optimal bid-acceptance strategy, for five different levels of outside price *p*



Note to Figure: An illustration of equation (7). The numbers next to the lines indicate the five levels of p. Each line reaches 1 at the respective $\beta(p)$. For bids between β^-_p and $\beta(p)$, the lines indicate the maximum acceptance probability such that no bidder bids at those levels.

Interestingly, the bid level $\beta(p)=1-\frac{1}{4p}$ at which the acceptance probability jumps to 1 is exactly the bid level at which $\lim_{x\to p}\beta(x;p)=\beta(x;p=1)$. This finding may contain deeper insight. Despite the apparent complexity of the implementation, the expected profit given *c* is just $\frac{(1-c)^2}{4}$.

6. Extension: Opacity of the NYOP offering

Suppose the NYOP seller makes the object description opaque in the sense of substituting a worthless object with probability λ. The outside posted-price offering remains clear (not opaque). In this setup, a natural question arises about the consumer's ability to dispose of the worthless object and either bid again or buy the outside posted-price offering. We use the simplifying assumption standard in the literature (e.g., Fay and Xie 2008) that the consumer has unit demand and there is no free disposal, so he consumes the worthless object.[4]

Let *x* be the valuation of the outside offering, distributed *F*(*x*) with support on $[\underline{x},\overline{x}]$ as in the non-opaque model. Also as before, assume the NYOP seller's cost continues to be distributed according to some *H* on [0,*p*], where *p* is the posted price of the outside offering. Deviating from the basic model, let *y* be the value of the opaque NYOP offering *net* of the outside option. For buyers with $x<p$ who do not have an outside option, the net value of the opaque NYOP offering is obviously $y=(1-\lambda)x$. For buyers with $x\geq p$, the net value of the opaque NYOP offering is $y=p-\lambda x$, which decreases with *x*, and does not exceed $(1-\lambda)p$. To see this interesting result, let the expected surplus of an $x>p$ buyer who bids *b* be

$$U(b,x)=A(b)\big(x(1-\lambda)-b\big)+\big[1-A(b)\big](x-p)=(x-p)+U(b,p-\lambda x) \qquad (8)$$

In other words, a "high" buyer with valuation $x>p$ bids the same amount as a "low" buyer with valuation $p-\lambda x<p$. The bid decreases in *x* because as *x* increases, so does the difference

[4] To allow for disposal and purchase of something else, we would have to model the consumer's budget constraint.

between the clear product available from the outside market and the opaque NYOP product. Note that the consumer with the highest effective valuation of the NYOP offering is the consumer with $x=p$, not a mass of consumers with $x \geq p$ as in the basic model.

Then the NYOP seller faces an effective distribution $\tilde{F}$ of y that combines both low-value bidders who are distributed $F\left(\frac{y}{1-\lambda}\right)$ on $\left[(1-\lambda)\underline{x},(1-\lambda)p\right]$, and high-value bidders who are distributed $1-F\left(\frac{p-y}{\lambda}\right)$ on $\left[p-\lambda\overline{x},(1-\lambda)p\right]$:

$$\tilde{F}(y)=\begin{cases} (1-\lambda)\underline{x} \leq y \leq p-\lambda\overline{x} : F\left(\frac{y}{1-\lambda}\right) \\ p-\lambda\overline{x} < y \leq p(1-\lambda) : 1-F\left(\frac{p-y}{\lambda}\right)+F\left(\frac{y}{1-\lambda}\right) \end{cases} \tag{9}$$

Therefore, the effective distribution has a kink (becomes steeper) at $p-\lambda\overline{x}$ and support on $\left[(1-\lambda)\underline{x},(1-\lambda)p\right]$. A higher density of bidders is present just above p-λ because for every "low" bidder with $x<p$, a "high" bidder with $x>p$ but the same net valuation y also exists. For example, when F is Uniform[0,1], the effective distribution of y is

$$\tilde{F}(y)=\begin{cases} 0 \leq y \leq p-\lambda : \frac{y}{1-\lambda} \\ p-\lambda < y \leq p(1-\lambda) : 1-\frac{p(1-\lambda)-y}{\lambda(1-\lambda)} \end{cases} \tag{10}$$

It is also useful to consider the implied virtual value $\tilde{\psi}(y) \equiv y-\frac{1-\tilde{F}(y)}{\tilde{f}(y)}$. The formulae are not particularly illuminating, but the highest virtual value is $\tilde{\psi}\left(p(1-\lambda)\right)=p(1-\lambda)$ for all F. In the Uniform[0,1] example, the virtual value reduces to

$$\tilde{\psi}(y)=\begin{cases} 0 \leq y \leq p-\lambda : 2y-(1-\lambda) \\ p-\lambda < y \leq p(1-\lambda) : 2y-p(1-\lambda) \end{cases} \tag{11}$$

The jump discontinuity at p-λ arises from the aforementioned kink in F. Not surprisingly, the formula reduces to the virtual value in the non-opaque model when λ=0 because that makes y=x, and the virtual value of a Uniform[0,1] x is $\psi(x)=2x-1$.

The basic intuition of Proposition 1 generalizes readily in that the NYOP seller would like to implement monopoly pricing given the effective demand $\tilde{F}$ he faces (proof omitted because it is analogous to the proof of Proposition 1):

Proposition 3: *When the NYOP seller offers an opaque product in that he substitutes a worthless product with probability λ, the optimal contingent bid-acceptance rule in a direct revelation mechanism is*

$$\pi(y,c)=\begin{cases}1 \text{ when } c<\tilde{\psi}(y) \\ 0 \text{ otherwise}\end{cases}$$

where $y=\min(x,p)-\lambda x$ *is the net value of the NYOP offering to a buyer who values the non-opaque outside-market offering x, and where* $\tilde{\psi}$ *is the virtual value of y. For every cost c, this rule achieves the same profit as a posted-price monopolist who knows c before setting his price.*

In contrast to the result in Proposition 1, the most valuable consumers are not ensured an NYOP acceptance with certainty, because the seller's cost is not guaranteed to be below their effective valuation. The highest net-value consumer (with x=p) will only get the good with probability $H\left[\tilde{\psi}\left(p(1-\lambda)\right)\right]<1$, because $\tilde{\psi}\left(p(1-\lambda)\right)=p(1-\lambda)<p$.

The NYOP implementation of the above $\pi(y,c)$ rule can be derived analogously to the derivation of the NYOP implementation in the basic model, and it is conceptually simpler because the effective distribution $\tilde{F}$ does not have a mass point. Consider a buyer with $y>\tilde{\psi}^{-1}(0)$ who follows a bidding strategy $\tilde{\beta}(y)$. His probability of winning in the optimal mechanism is $q(y)=H\left(\tilde{\psi}(y)\right)$, and his expected utility is thus $U(y)=\int_{\underline{y}}^{y} H\left(\psi(t)\right)dt$. Utilizing the revenue equivalence between the optimal direct-revelation mechanism and the NYOP mechanism, the bidding strategy $\tilde{\beta}(y)$ must satisfy: $\tilde{\beta}(y) = y - \frac{U(y)}{q(y)}$.

Note that although the bidding function is obviously increasing in *y*, it is decreasing in *x* when *x*>*p*, because a buyer with valuation *x*>*p* bids the same amount as a buyer with valuation $p - \lambda x < p$.

6.1 Example: Opacity of NYOP with *F* and *H* Uniform

Assume $2p - \lambda - 1 > 0$, so the optimal monopoly price given *c*=0 is $\frac{(1-\lambda)}{2}$. Then *F* and *H* uniform imply

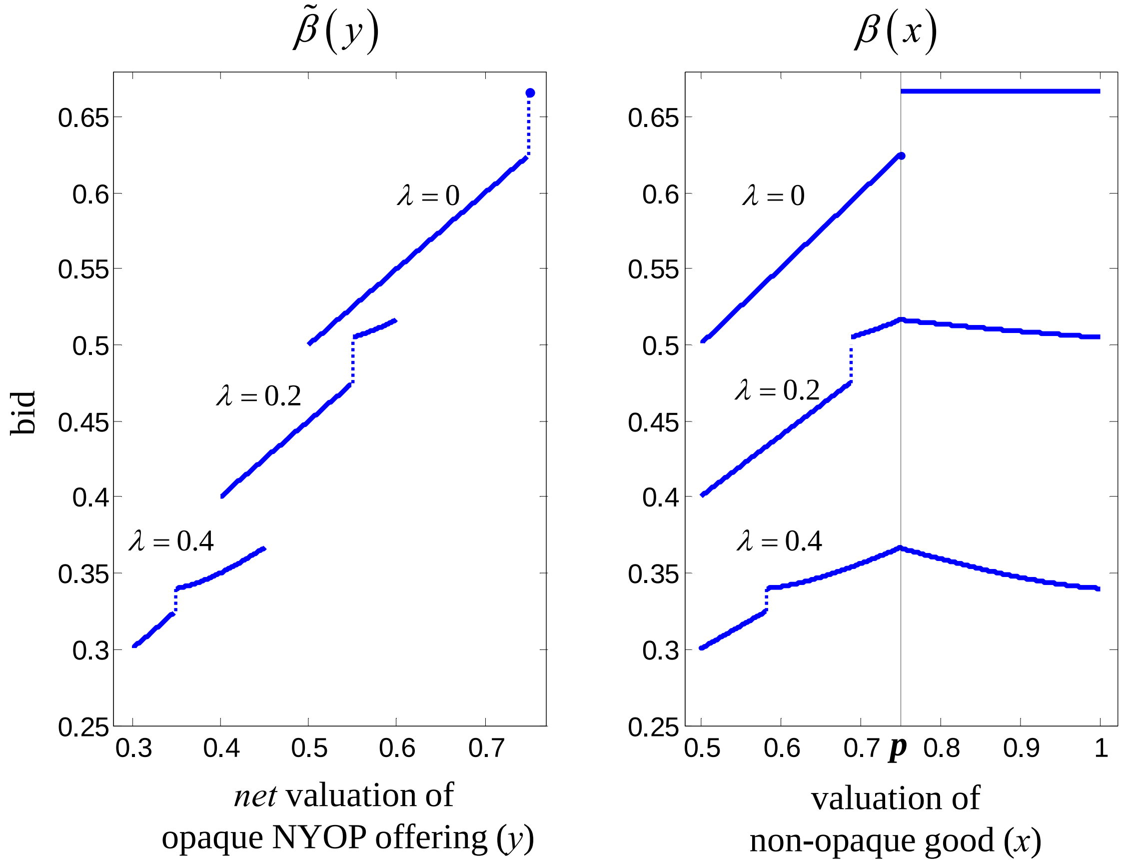
$$\tilde{\beta}(y) = \begin{cases} \frac{(1-\lambda)}{2} \le y \le p - \lambda : \frac{y}{2} + \frac{1-\lambda}{4} \\ p - \lambda < y \le p(1-\lambda) : \frac{4y^2 - (1-\lambda)\left[\lambda(3-4p) + (1-2p)^2\right]}{8y - 4p(1-\lambda)} \end{cases} \tag{12}$$

where the second expression is made complicated by the discontinuity in $\tilde{\psi}$. Changing variables from *y* back to the underlying valuation *x* yields

$$\beta(x) = \begin{cases} \frac{1}{2} \le x \le \frac{p-\lambda}{1-\lambda} : \frac{x(1-\lambda)}{2} + \frac{1-\lambda}{4} = (1-\lambda)\beta(x \mid \lambda = 0) \\ \frac{p-\lambda}{1-\lambda} < x \le p : \frac{4x^2(1-\lambda) - \left[\lambda(3-4p) + (1-2p)^2\right]}{8x - 4p} \\ p < x \le 1 : \frac{4(p-\lambda x)^2 - (1-\lambda)\left[\lambda(3-4p) + (1-2p)^2\right]}{8(p-\lambda x) - 4p(1-\lambda)} \end{cases} \tag{13}$$

It is easy to confirm that substituting λ=1 leads to the bidding function from the model without opacity. Figure 3 shows the bidding strategy both in terms of the net valuation of the NYOP offering and the underlying valuation.

Figure 3: NYOP bidding functions that implement the optimal mechanism, for three levels of opacity of the NYOP offering.



Note to Figure: The outside posted price is 0.75 throughout. Dashed vertical lines indicate the jump discontinuities in the bidding functions.

7. Extension: Strategies available to sellers with limited commitment

The amount of commitment required for fully optimal NYOP selling may be too strong for some real-world markets. In this extension, we consider three types of "limited commitment" sellers: first, sellers may be able to commit to only considering bids above some minimum level, but not to a probability of bid acceptance above that level. Such sellers can effectively set a minimum bid akin to a public reserve in an auction.

Second, the sellers may be able to charge a participation fee, that is, ask to be paid for considering bids, as proposed by Spann, Zeithammer, and Haübl (2010).

Finally, we want to analyze the case of a "passive" seller with no commitment beyond the ability to credibly reject unprofitable bids (bids below cost). The profits available to such a seller are the relevant baseline from which all of the above sellers improve. Considering the profits of such a non-commitment seller also isolates the role of *p* in profitability.

7.1 Participation fee

Spann, Zeithammer, and Haübl (2010, 2013) assume *F*=Uniform[0,1] and *H*=Uniform[0,*p*], and show that the seller profits more from charging a participation fee than from charging a minimum markup or from some combination of a participation fee and a minimum markup. The optimal fee *e* to charge is (Proposition 2 of Spann, Zeithammer, and Haübl 2013)

$$\sqrt{e^*(p)} = \min\left(\frac{2}{7\sqrt{p}}, \frac{\sqrt{p}}{2}\right) = \begin{cases} \frac{\sqrt{p}}{2} & \text{for } p < \frac{4}{7} \approx 0.56 \\ \frac{2}{7\sqrt{p}} & \text{for } p \geq \frac{4}{7} \approx 0.56 \end{cases} \tag{14}$$

and the resulting seller profit is:

$$\Pi(e^*) = \begin{cases} \frac{3p(1-p)}{8} & \text{for } p < \frac{4}{7} \approx 0.56 \\ \frac{4}{147p} + \frac{p}{8} - \frac{p^2}{12} & \text{for } p \geq \frac{4}{7} \approx 0.56 \end{cases}$$

The participation fee screens low-valuation buyers out of the market, and the implied entry threshold is $2\sqrt{pe^*} = \min\left(\frac{4}{7}, p\right)$. The seller uses the rule that maximizes his *c*-contingent ex-post profit, namely, $sell \Leftrightarrow b > c$, so $A(b) = \Pr(b > c) = H(b)$, so the buyers who enter bid $\beta(x) = \frac{x}{2}$. Therefore, the full allocation rule is

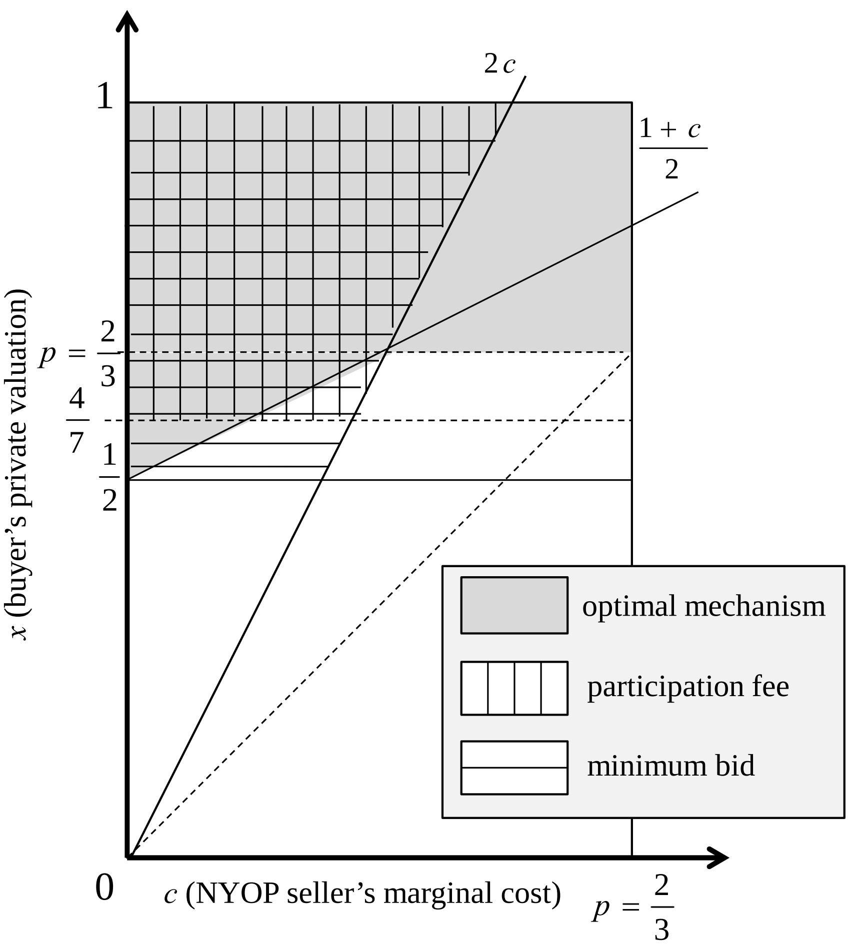
$$q_e(x) = \begin{cases} 0 & \text{for } x \in [0, 4/7) \\ x/2 & \text{for } x \in [4/7, 1] \end{cases} \tag{15}$$

To rank seller profits in full-commitment and participation-fee selling, it is enough to compare the allocation rules. Figure 4 shows the two allocation rules. It is clear that the optimal mechanism is more efficient, especially for higher seller costs. Specifically, the optimal mechanism-allocation rule is

$$q(x)=\begin{cases}0 \text{ for } x\in\left[\underline{x},\psi^{-1}(0)\right]\\ H(\psi(x)) \text{ for } x\in\left[\psi^{-1}(0),p\right)\\ 1 \text{ for } x\in\left[p,\overline{x}\right]\end{cases}\overset{H \text{ and } F}{\underset{uniform}{=}}\begin{cases}0 \text{ for } x\in[0,1/2]\\ 2x-1 \text{ for } x\in[1/2,p)\\ 1 \text{ for } x\in[p,1]\end{cases} \tag{16}$$

Because $q_e(x)\neq q(x)$, no participation fee can implement the optimal mechanism.

Figure 4: Allocation rules: the optimal mechanism vs. optimal participation fee



Note to Figure: The posted price *p* is set to $2/3$ throughout the figure. The support of (x,c) is in the *p*x1 rectangle, and *F*(*x*) is uniform. The shaded area indicates the allocation rule of the optimal mechanism. The area filled by vertical lines indicates the allocation rule of the optimal participation fee. Note that all consumers with $x > 4/7$ enter and pay the participation fee.

7.2 Minimum bid

The implementation of the optimal mechanism requires seller commitment to a particular probabilistic bid-acceptance strategy $A(b) \equiv \Pr(accept\ b)$, which sometimes rejects ex-post profitable bids ($b>c$) and sometimes accepts ex-post unprofitable bids ($b<c$). Suppose instead that the NYOP seller cannot commit to accepting or rejecting bids as a function of c, but can credibly refuse to consider bids below a certain minimum level m. Once he considers a bid, he follows the rule that maximizes his c-contingent ex-post profit, namely, $A(b)=H(b)$.

Commitment to a minimum bid is easier than commitment to an arbitrary $A(b)$, because it is a commitment to a pure strategy, and hence can be verified on a case-by-case basis. Moreover, a minimum bid can also be credible via common knowledge that getting cost quotes from suppliers is costly (a low bid is then not worth considering because it does not cover the cost of a supplier quote in expectation). Finally, commitment to a minimum bid is clearly realistic because it is the same type of commitment as that required for using a public reserve in auctions—a commonly observed feature in most auction markets.

Another reason to analyze the minimum-bid strategy is the fact that minimum bids are a salient feature of the fully optimal policy. It is interesting to investigate how much of the profit-increasing potential of the fully optimal policy is achieved via the minimum bid alone, and how the added profit due to full commitment depends on the outside market price p.

We now solve for the optimal minimum-bid strategy. Buyers with $x<m$ do not enter the NYOP market, because they cannot earn a non-negative surplus. Buyers with $x \geq m$ solve $\beta(x) = \arg\max_{b \geq m} H(b)(x-b)$. Let b^* solve the first-order condition of the unconstrained problem, namely, $b^* = x - \frac{H(b^*)}{h(b^*)}$, and assume H satisfies the usual conditions for b^* to be the argmax of $H(b)(x-b)$. Then the buyers' best response to m is to bid $\beta(x) = \max(b^*, m)$. In other words, a mass of buyers with valuations just above m all pool on bidding m.

Assume for tractability (and for a direct comparison with the participation-fee result) that *H* is uniform on [0,*p*]. Then $b^* = \frac{x}{2}$ and $\beta(x) = \max\left(\frac{x}{2}, m\right)$. In other words, buyers with $x \in [m, 2m]$ all bid *m*. The seller profit is often increasing in *m* for *m*<*p*/2 : in the Appendix, we show that $\frac{d\Pi}{dm} > 0 \Leftrightarrow 2\left[F(2m) - F(m)\right] > mf(m)$, which holds, for example, for the Uniform *F* distribution. For *m*>*p*/2, every buyer who enters the market bids *m,* simplifying the profit to

$$\Pi(m) = \left[1 - F(m)\right] \int_0^p \mathbf{1}(m > c)(m - c)\, d\left(\frac{c}{p}\right) = \left[1 - F(m)\right] \frac{m^2}{2p} \tag{17}$$

Maximizing the profit is isomorphic to monopoly pricing with zero marginal cost:

$$m = \frac{1 - F(m)}{f(m)} \Leftrightarrow m = \psi^{-1}(0)$$

When *F* is uniform, $m^* = \frac{1}{2}$; thus $\Pi(m^*) = \frac{1}{16p}$. We have shown the following:

Proposition 5*: When H is uniform on* $[0, p]$ *and* $2\left[F(2m) - F(m)\right] > mf(m)$ *for all* $m \leq p/2$*, the minimum bid used by a limited-commitment seller is the monopoly price the seller would charge if his marginal cost were zero, namely, the minimum bid used by the seller with full commitment.*

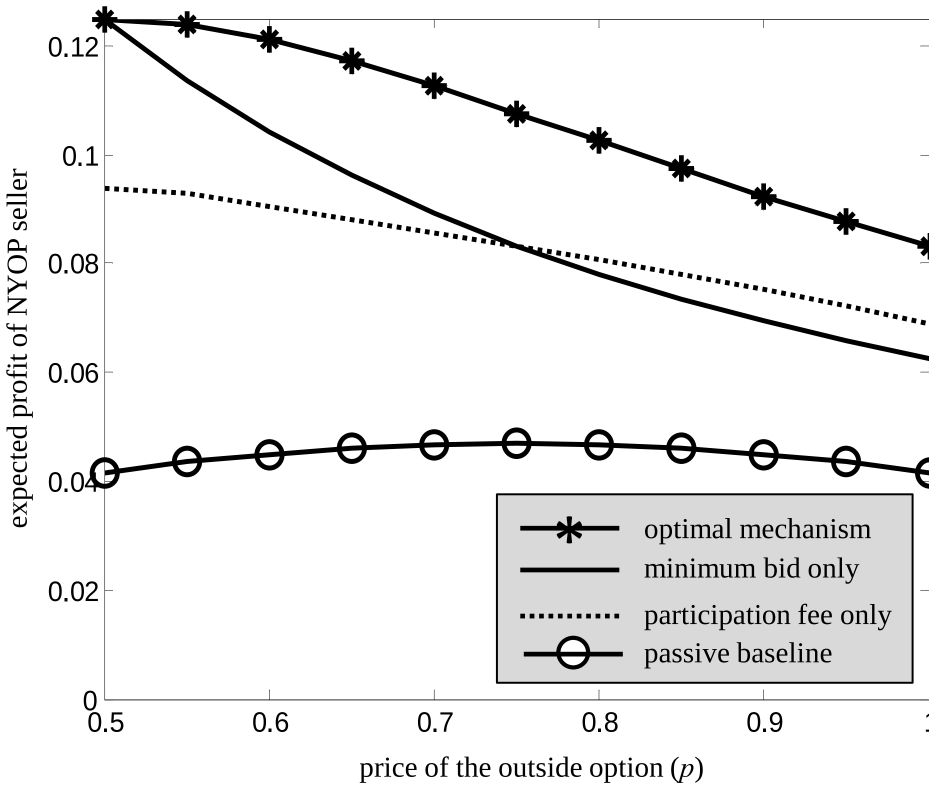
7.3 Passive seller

When seller is passive (i.e., cannot commit to anything and cannot charge a participation fee) and hence accepts all bids above cost, he makes the profit

$$\Pi = \int_0^p \int_0^p \mathbf{1}\left(\frac{x}{2} > c\right)\left(\frac{x}{2} - c\right) d\left(\frac{c}{p}\right) dx + (1 - p) \int_0^p \mathbf{1}\left(\frac{p}{2} > c\right)\left(\frac{p}{2} - c\right) d\left(\frac{c}{p}\right) = \frac{p(3 - 2p)}{24} \tag{18}$$

Note that this profit is hill shaped, and maximized at *p*=0.75. This non-monotonicity demonstrates how a lower outside-market price is both bad news (tougher competition) and good news (lower expected cost) for the NYOP seller. The competitive effect is stronger for low *p*, the cost-reduction effect for high *p*.

Figure 5: Expected profit of NYOP sellers with four different levels of commitment



7.4 Profit comparison of limited-commitment sellers

Figure 5 plots all four (full commitment, two limited-commitment, and passive) profits as a function of the price of the outside option, and illustrates our last proposition that compares the two limited-commitment sellers:

Proposition 6: *For any F and H, as p approaches* $\psi^{-1}(0)$ *from above, the relative profit advantage of full commitment over limited commitment to a minimum bid vanishes. When F is uniform on* $[0,1]$ *and H is uniform on* $[0,p]$, *a price* $\frac{3}{4} < p^* < \frac{4}{5}$ *exists such that commitment to a participation fee is more profitable than commitment to a minimum bid for all p>p*.*

To gain intuition for the first point in Proposition 6, note that when p= ½, both the fully optimal contingent bid-acceptance rule and the optimal minimum bid strategy reduce to posted pricing: the posted price is m*= ½, and anyone willing to pay it receives the object with certainty.

To gain intuition for the second point of Proposition 6, note that participation fees are always strictly suboptimal, so the first bullet implies $\Pi\left(e^*\middle| p=\frac{1}{2}\right)<\Pi\left(m^*\middle| p=\frac{1}{2}\right)$. However, the seller with commitment to a participation fee is better able to extract profit as p increases. To see why, note that p does not affect entry and bidding when a seller commits to a minimum bid: buyers with $x>m^*=\frac{1}{2}$ enter and bid m, regardless of p. Therefore, the profit $\Pi\left(m^*\middle| p\right)$ is decreasing in p purely due to rising seller costs. The participation-fee seller faces the same rising costs, but p also increases his buyers' bids because the bidding strategy is $\frac{\min\left(x,p\right)}{2}$. Therefore, $\Pi\left(e^*\middle| p\right)$ decreases more slowly than $\Pi\left(m^*\middle| p\right)$.

8. Discussion

Name-your-own-price (NYOP) selling accommodates buyer activism whereby buyers submit bids for products the seller may or may not be able to procure at a reasonable cost. We show that an NYOP seller can achieve first-best ex-post profits despite not knowing his cost realization at the time of announcing his strategy. Specifically, we analyze a market in which the seller announces a bid-acceptance probability schedule, the buyers best respond with their bids, and only after the bid is received does the seller learn his cost. We show how to craft the bid-acceptance probability schedule in order for the seller to make as much profit as he would if he could learn his cost first and use the optimal mechanism contingent on it (it is well known that the contingent optimal mechanism is to make a take-it-or-leave-it offer at a fixed price).

As long as the distribution of valuations is regular, the buyers respond with an increasing bidding function that allows the seller to implement the optimal mechanism contingent on cost.

The seller can invert the bids and only accept bids of buyers with high-enough valuations, that is, valuations above the ex-post optimal monopoly price. Seemingly, the seller could use any increasing bid-acceptance probability to achieve his goal of inverting the bidding function, but only the bid-acceptance probability implied by the ex-post monopoly allocation is sustainable in equilibrium (buyers are not deceived).

The implementability result is robust to an arbitrary price in the outside posted-price market and to opacity of the NYOP offering. These realistic features change both the bid-acceptance probability and its associated bidding function, but they do not hinder implementation. When the NYOP offering is not opaque, the outside posted-price market is a real option for high-valuation buyers, who mimic the buyer with valuation equal to the outside price. This pooling leads to a jump discontinuity in the equilibrium bidding function. When the NYOP offering is not opaque, pooling by high-valuation buyers does not occur because their *net* valuations of the NYOP offering (and hence their bids) are *de*creasing in their valuations. The bidding function continues to exhibit a jump discontinuity because each high-valuation buyer mimics a particular medium-valuation buyer. In summary, the bidding functions that implement the optimal mechanism can be non-intuitive. Correspondingly, surprising aspects of the optimal bid-acceptance policy arise, which we discuss next.

The optimal bid-acceptance probability requires both commitment to reject potentially profitable low bids (e.g., all bids below the minimum bid) and commitment to subsidize some unprofitable high bids (e.g., bids by high-value bidders who all bid as if their valuation were equal to the outside posted price). The latter type of commitment is new in that it is not required for standard monopoly pricing.

The amount of commitment required for full optimality may be too strong for some real-world markets. In this extension, we consider three types of "limited commitment" sellers: first, sellers may be able to commit to considering bids only above some minimum level, but not to a probability of bid acceptance above that level. Second, the sellers may be able to charge a participation fee, that is, ask to be paid for considering bids, as proposed by Spann, Zeithammer, and Haübl (2010). Finally, the sellers may be completely passive and have no commitment beyond the ability to credibly reject unprofitable bids (bids below cost). We find that the relative profitability of the two limited-commitment strategies depends on the level of the outside price: when the outside price is high, the participation fee dominates the minimum bid, and vice versa. The minimum-bid strategy is good at raising everyone's bid—useful when the outside option is attractive to many buyers. Conversely, the bidding subsequent to paying a participation fee is better at price discriminating—useful when pooling by high-valuation buyers is low. We also show that the relative advantage of full commitment over minimum-bid-only vanishes when the outside price is low, and the fully optimal strategy boils down to posted pricing.

The implementability of the optimal mechanism via NYOP selling can be interpreted as an extension of the well-known revenue equivalence between first-price and second-price auctions to a world with a single bidder and uncertain seller costs: the optimal direct-revelation mechanism is equivalent to a set of second-price sealed-bid auctions, each with an optimal reserve price. The insight of this paper is to notice that NYOP selling corresponds to a first-price sealed-bid auction with a single bidder (the buyer) and a stochastic secret reserve (the seller's bid-acceptance function), which is constructed to accept a bid exactly often enough to maintain revenue equivalence with the corresponding second-price auction. In light of revenue equivalence between first-price and second-price sealed-bid auctions, the revenue equivalence between NYOP and posted pricing is thus less surprising.

Regarding practical implementation, the mechanism proposed here is more in line with "select your price" than with true "name your own price" in Chernev (2003). Moreover, we suggest the seller should present not only a menu of prices, but also the acceptance probabilities. Doing so facilitates commitment and simplifies bidding. Outside of the scope of this paper, it also enables the seller to learn consumer preferences better by controlling consumer beliefs about chances of bid acceptance.

References

Anderson, Chris. 2009. Setting Prices on Priceline. *Interfaces* 39(4), 307–315.

Amaldoss and Jain. 2008. Joint Bidding in the Name-Your-Own-Price Channel: A Strategic Analysis, *Management Science*, Vol. 54, No. 10, pp. 1685-1699.

Becker Gordon M., Moriss H. DeGroot and Jacob Marschak. 1964. Measuring utility by a single-response sequential method. *Behavioral Science* 9 (3), 226–32.

Chernev, Alexander. 2003. Reverse Pricing and Online Price Elicitation Strategies in Consumer Choice. *Journal of Consumer Psychology*, 13, 51-62.

Ding, Min, Jehoshua Eliashberg, Joel Huber, and Ritesh Saini. 2005. Emotional Bidders: An Analytical and Experimental Examination of Consumers' Behavior in a Priceline-Like Reverse Auction. *Management Science*, 51:3, 352-364.

Fay, Scott. 2004. Partial-Repeat-Bidding in the Name-Your-Own-Price Channel. *Marketing Science* 23:3, 407-418.

Fay, Scott. 2008. Selling an Opaque Product through an Intermediary: The Case of Disguising One's Product", *Journal of Retailing*, Vol. 84 (1), pp. 59 - 75.

Fay, Scott and Jinhong Xie. 2008. Probabilistic Goods: An Innovative Way of Selling Products and Services. *Marketing Science* 27(4), 674-690.

Fay, Scott. 2009. Competitive reasons for the Name-Your-Own-Price channel. *Marketing Letters* v. 20. pp. 277-293.

Hann, Il-Horn and Christian Terwiesch. 2003. Measuring the Frictional Costs of Online Transactions: The Case of a Name-Your-Own-Price Channel." *Management Science*. 49 (11) 1563-1579.

Krishna, Vijay. 2002. *Auction Theory*. London: Academic Press of Elsevier Science.

Myerson, Roger B. 1981. Optimal Auction Design. *Mathematics of Operations Research* 6(1), 58-73.

Riley, John and Richard Zeckhauser. 1983. Optimal Selling Strategies: When to Haggle, When to Hold Firm. *Quarterly Journal of Economics*, 98:2, 267-289.

Shapiro, Dmitry. 2011. Profitability of Name Your Own Price Mechanisms in the Case of Risk-Averse Buyers, *Marketing Science* 30(2), 290-304.

Spann, Martin and Gerard J. Tellis. 2006. Does the Internet Promote Better Consumer Decisions? The Case of Name-Your-Own-Price Auctions, *Journal of Marketing*, Vol. 70, No. 1, pp. 65-78, January 2006

Shapiro Dmitry and Artie Zillante. 2009. Naming Your Own Price Mechanisms: Revenue Gain or Drain? *Journal of Economic Behavior and Organization*, Vol. 72, pp. 725-737.

Shapiro Dmitry and Shi, Xianwen. 2008. Market Segmentation: The Role of Opaque Travel Agencies, *Journal of Economics & Management Strategy* 17(4), pp. 803-837.

Spann, Martin, Gerald Haübl , Bernd Skiera, and Martin Bernhardt. 2012. Bid-Elicitation Interfaces and Bidding Behavior in Retail Interactive Pricing. *Journal of Retailing* 88(1), 131–144.

Spann, Martin, Robert Zeithammer, and Gerald Haübl, 2010, Optimal Reverse-Pricing Mechanisms, *Marketing Science* 29(6), 1058-1070.

Spann, Martin, Robert Zeithammer, and Gerald Haübl, 2013, Additional Results on Optimal Reverse Pricing, *working paper*.

Vickrey, William. 1961. Counterspeculation, Auctions, and Competitive Sealed Tenders. *Journal of Finance* 16 (1): 8–37.

Wang, Tuo and Esther Gal-Or and Rabikar Chatterjee. 2009. The Name-Your-Own-Price Channel in the Travel Industry. *Management Science* 55(6): 968-979.

Appendix

Table 1: Notation

x : buyer's valuation of the object
b : buyer's bid
$f(x), F(x)$: density and the c.d.f. the distribution of x
$\psi(x) \equiv x - \frac{1-F(x)}{f(x)}$: virtual value of type x.
c : seller's wholesale for of the object
$h(c), H(c)$: density and the c.d.f. the distribution of c
p : lowest price posted in an outside spot market for the object
$\beta(x)$: buyer's bidding function
$A(b)$: probability that seller accepts bid b
$U(b,x)$: expected surplus of a buyer x who bids b
$q(x)$: ex-ante probability that seller allocates the object to a buyer with valuation x
$\pi(x,c)$: probability that seller with cost c allocates the object to a buyer with valuation x
λ : opacity of the NYOP offering, i.e., probability of an inferior product (in Extension 1)
y : buyer's *net* valuation of the NYOP opaque offering (in Extension 1)
$\tilde{f}(y), \tilde{F}(y)$: density and the c.d.f. the distribution of y (in Extension 1)
e : participation fee (in Extension 2)
m : minimum bid (in Extension 2)

Proof of Proposition 1: Let $m(x)$ be the expected payment by a buyer with valuation x. From risk neutrality, the utility of a buyer x who reports type z is $U(x) = xq(z) - m(z)$, and standard incentive-compatibility arguments (see Myerson 1981 for details) imply that the expected payment is the following function of $q(x)$:

$$m(x) = m(0) + xq(x) - \int_0^x q(t)\,dt \qquad \text{(IC)}$$

When (IC) does not hold, the buyers do not have the incentive to report their x truthfully.

Consider the direct-revelation seller with commitment who can set an arbitrary bid-acceptance rule $\pi(x,c)$. Plugging the implied bid-acceptance rule $q(x)=\int_0^p \pi(x,c)\,dH(c)$ into (IC) implies that *on average over all c*, such a seller receives a payment of

$$m(x)=m(0)+x\int_0^p \pi(x,c)\,dH(c)-\int_{\underline{x}}^{x}\int_0^p \pi(t,c)\,dH(c)\,dt \tag{A1}$$

Note the rule π can use c as an input, so the seller can set the rule for all possible c levels upfront. However, the buyers do not know c at the time of submitting their bids, so incentive compatibility only restricts the average payment of a given buyer type. Because all buyers with $x \geq p$ pay $m(p)$, the expected profit of the seller is

$$\begin{aligned}\Pi(\pi)=&\Pr(x<p)\left(E_{x|x<p}\left[m(x)\right]-E_{c,x|x\leq p}\left[c\pi(x,c)\right]\right)\\&+\left[1-\Pr(x<p)\right]\left[m(p)-E_c\left[c\pi(p,c)\right]\right]\end{aligned} \tag{A2}$$

Plugging the m function from (A1) into the profit expression (A2) yields

$$\begin{aligned}\Pi(\pi)=&m(0)+\int_{\underline{x}}^{p} x\int_0^p \pi(x,c)\,dH(c)\,dF(x)-\int_{\underline{x}}^{p}\int_0^x\int_0^p \pi(t,c)\,dH(c)\,dt\,dF(x)-\int_{\underline{x}}^{p}\int_0^p c\pi(x,c)\,dH(c)\,dF(x)\\&+\left[1-F(p)\right]\left[p\int_0^p \pi(p,c)\,dH(c)-\int_0^p\int_0^p \pi(t,c)\,dH(c)\,dt-\int_0^p c\pi(p,c)\,dH(c)\right]\end{aligned} \tag{A3}$$

where the second row corresponds to the profit from high buyers ($x \geq p$), and the last term in each row is the expected cost of goods sold.

As in other mechanism-design settings, the term $\int_{\underline{x}}^{p}\int_0^x\int_0^p \pi(t,c)\,dH(c)\,dt\,dF(x)$ in the first row can be simplified by first changing the order of integration from *c,t,x* to *x,t,c*, and noting that $\pi(t,c)$ does not depend on *x*:

$$\begin{aligned}&\int_{\underline{x}}^{p}\int_0^x \pi(t,c)\,dt\,dF(x)=\int_0^p\left(\pi(t,c)\int_t^p dF(x)\right)dt=\\&=\int_0^p \pi(t,c)\left[F(p)-F(t)\right]dt=\int_{\underline{x}}^{p} \pi(x,c)\left(\frac{F(p)-F(x)}{f(x)}\right)dF(x)\end{aligned} \tag{A4}$$

where the last equality simply renames the *t* variable as *x* and changes variables. Finally, change the order of integration to be first over *x* and then over *c* throughout, and collect terms:

$$\Pi(\pi) = m(0) + \int_0^p \int_{\underline{x}}^p \left(x - \frac{F(p) - F(x)}{f(x)} - c \right) \pi(x,c)\, dF(x)\, dH(c) + \\ + [1 - F(p)] \left[\int_0^p (p-c)\pi(p,c)\, dH(c) \right] - [1 - F(p)] \int_0^p \int_{\underline{x}}^p \pi(x,c)\, dx dH(c) \tag{A5}$$

The last term in (A5) is the expected surplus of the high buyers ($x \geq p$). It obviously depends on the allocation rule for all $x \leq p$, and after rewriting it as $\int_0^p \int_{\underline{x}}^p \pi(x,c) \left(\frac{1-F(p)}{f(x)} \right) dF(x)\, dH(c)$, we can incorporate it into the first row of (A5) to result in[5]

$$\Pi(\pi) = E_c \left[\int_{\underline{x}}^p \left(x - \frac{1-F(x)}{f(x)} - c \right) \pi(x,c)\, dF(x) + [1 - F(p)](p-c)\pi(p,c) \right]. \tag{A6}$$

Equation (A6) implies the seller profit is *as if* all high customers paid *p* and all low customers delivered the same profit they would in the absence of the posted-price competitor. In other words, the surplus of high-value buyers implied by the IC constraint affects the payments of low-value buyers exactly as it would in the absence of the posted-price competitor.

The optimal allocation rule is obvious, and it maximizes the expected profit pointwise:

$$\pi(x,c) = 1 \Leftrightarrow \left(x < p \text{ and } c < x - \frac{1-F(x)}{f(x)} \right) \text{ or } x = p \tag{A7}$$

To see the optimality of always selling to high-value buyers with $x=p$, note that although the term $\pi(p,c)$ appears in (A6) twice, its impact on profits inside the integral is measure zero, whereas its impact on profits in the $[1-F(p)](p-c)\pi(p,c)$ term has positive measure.

To derive the *c*-contingent profit shown in the proposition, use integration by parts to show that $[1-F(p)](p-c) = \int_p^{\bar{x}} (\psi(x) - c)\, dF(x)$, and so plugging A7 into the term inside the square bracket in A6 yields $\int_{\psi^{-1}(c)}^{\bar{x}} (\psi(x) - c)\, dF(x)$.   *QED Proposition* 1

[5] Standard individual rationality arguments also imply $m(0) = 0$.

Proof of Lemma 1: Because $q(x)=U'(x)$, $\beta'(x)=\dfrac{U(x)U''(x)}{\left(U'(x)\right)^2}>0 \Leftrightarrow U''(x)>0 \Leftrightarrow q'(x)>0$.

From Proposition 1, $q'(x)=\psi'(x)h(\psi(x))>0 \Leftrightarrow \psi'(x)>0$. To derive the optimal bidding function, plug q and U from Proposition 1 into equation 2:

$$\beta(x)=\int_{\psi^{-1}(0)}^{x} t\frac{dH(\psi(t))}{H(\psi(x))}=\int_{0}^{\psi(x)}\psi^{-1}(c)\frac{dH(c)}{H(\psi(x))}=E\left[\psi^{-1}(\text{cost})\middle|\psi^{-1}(\text{cost})<x\right],$$

where the second equality follows from a change in variables $c=\psi(z)$.

Deriving the bidding function directly is also possible: Suppose the seller inverts the bids he receives to find the buyer's true valuation x and sells whenever $c<\psi(x)$. The bidding strategy in response to such an allocation solves

$$\max_z H\left(\psi\left(\beta^{-1}(z)\right)\right)(x-z)$$

The first term is the probability that the seller accepts the bid, that is, the probability that c is low enough. The first-order condition is

$$h\left(\psi\left(\beta^{-1}(z)\right)\right)\psi'\left(\beta^{-1}(z)\right)(x-z)=\beta'\left(\beta^{-1}(z)\right)H\left(\psi\left(\beta^{-1}(z)\right)\right) \qquad \text{(FOC)}$$

In equilibrium, $z=\beta(x)$, so the FOC becomes

$$h(\psi(x))\psi'(x)x=\beta'(x)H(\psi(x))+\beta(x)h(\psi(x))\psi'(x)=G'(x)$$

where $G(x)\equiv\beta(x)H(\psi(x))$. Solving for $G(x)$ is simple:

$$G(x)=const+\int_{\psi^{-1}(0)}^{x} zh(\psi(z))\psi'(z)\,dz=const+\int_{0}^{\psi(x)}\psi^{-1}(w)h(w)\,dw$$

where the second equality follows from a change in variables $w=\psi(z)$. The definition of $G(x)$ together with $\beta\left(\psi^{-1}(0)\right)=\psi^{-1}(0)$ implies the optimal bidding strategy:

$$\beta(x)=\int_{0}^{\psi(x)}\psi^{-1}(w)\frac{h(w)}{H(\psi(x))}dw=E\left[\psi^{-1}(c)\middle|\psi^{-1}(c)<x\right] \qquad \textit{QED Lemma } 1$$

Proof of Lemma 2: To be incentive compatible, $\beta(p)$ must satisfy, for every *x*<*p*:

$$p-\beta(p)\geq A(\beta(x))(p-\beta(x)) \qquad (IC1)$$
$$U(x)\geq x-\beta(p) \qquad (IC2)$$

where the first inequality $(IC1)$ ensures type *p* bids $\beta(p)$, and the second inequality $(IC2)$ ensures types *x*<*p* do not deviate to $\beta(p)$. The deviation surplus for type *p* is

$$A(\beta(x))(p-\beta(x))=H(\psi(x))\left(p-\int_0^{\psi(x)}\psi^{-1}(w)\frac{h(w)}{H(\psi(x))}dw\right)=\int_0^{\psi(x)}\left(p-\psi^{-1}(w)\right)dH(w)$$

which is obviously increasing in *x*, so the best deviation from bidding *p* is to bid β_p^-. Therefore, $(IC1)$ reduces to $\beta(p)\leq p-\int_0^{\psi(p)}\left(p-\psi^{-1}(w)\right)dH(w)$. The LHS of $(IC2)$ is the expected equilibrium surplus of type *x*, which Proposition 1 pins down as $U(x)=\int_0^x H(\psi(t))dt$.

Therefore, $(IC2)$ is $\beta(p)\geq p-\int_0^{\psi(p)}\left(p-\psi^{-1}(w)\right)dH(w)$ because $x-\int_0^{\psi(x)}\left(x-\psi^{-1}(w)\right)dH(w)=x-\int_{\psi^{-1}(0)}^{x}H(\psi(z))dz$ is increasing in *x*. Therefore, the two incentive-compatibility constraints uniquely determine the bid of type *p* as

$$\beta(p)=p-\int_0^{\psi(p)}\left(p-\psi^{-1}(w)\right)dH(w)=p-\int_{\psi^{-1}(0)}^{p}H(\psi(z))dz$$

It is easy to show that the invertibility constraint $\beta(p)>\beta_p^-$ is always satisfied. *QED Lemma* 2

Proof of Lemma 3: The upper bound on probability acceptance must simply satisfy, for every *x*,

$$A(b)(x-b)\leq H(\psi(x))(x-\beta(x))$$

The RHS is maximized by *x*=*p*, so using the same arguments as in the proof of Lemma 2, the constraint is thus $A(b)\leq\left(\frac{1}{p-b}\right)\int_{\psi^{-1}(0)}^{p}H(\psi(z))dz$ *QED Lemma* 3

Proof of Proposition 6: To prove the first bullet, note that $\Pi^*\left(\frac{1}{2}\right)=\frac{1}{8}=\Pi\left(m^*\middle|p=\frac{1}{2}\right)$. To prove the second bullet, let $\Delta\Pi(p)=2352p\left[\Pi(e^*|p)-\Pi(m^*|p)\right]=98p^2(3-2p)-83$.

Because $\frac{d\Delta\Pi}{dp}=588p(1-p)>0$, $\Delta\Pi(p)$ is increasing in *p* on the interval $\left[\frac{4}{7},1\right]$.

Because $\Delta\Pi\left(\frac{3}{4}\right)=-\frac{5}{16}<0<\frac{601}{125}=\Delta\Pi\left(\frac{4}{5}\right)$, $\Delta\Pi(p)$ has a root $\frac{3}{4}<p^*<\frac{4}{5}$. It is easy to show $\Pi(e^*|p)<\Pi(m^*|p)$ on the remainder of the support of *p*, namely, $p\in\left[\frac{1}{2},\frac{4}{7}\right]$. *QED Prop* 6

Details of derivations in the Second Extension section

The expected profit of a seller who uses a participation fee *e* is

$$\Pi(e)=\Pr(\underline{c}<c)e+E_{c,v>\underline{v}}\left[\mathbf{1}(b>c)(b-c)\right]=$$
$$=\left[1-F(\underline{x})\right]e+\int_{\underline{x}}^{p}\int_{0}^{p}\mathbf{1}\left(\frac{x}{2}>c\right)\left(\frac{x}{2}-c\right)d\left(\frac{c}{p}\right)dF(x)+\left[1-F(p)\right]\int_{0}^{\frac{p}{2}}\left(\frac{p}{2}-c\right)d\left(\frac{c}{p}\right)$$

Substituting the optimal fee $\sqrt{e^*}=\min\left(\frac{2}{7\sqrt{p}},\frac{\sqrt{p}}{2}\right)$ and the implied entry threshold is $2\sqrt{pe^*}=\min\left(\frac{4}{7},p\right)$ yields:

$$\Pi(e^*)=\begin{cases}\frac{3p(1-p)}{8} & \text{for } p<\frac{4}{7}\approx 0.56\\ \frac{4}{147p}+\frac{p}{8}-\frac{p^2}{12} & \text{for } p\geq\frac{4}{7}\approx 0.56\end{cases}$$

The expected profit of a seller who uses a minimum bid is

$$\Pi(m)=\left[F(2m)-F(m)\right]\int_{0}^{p}\mathbf{1}(m>c)(m-c)d\left(\frac{c}{p}\right)+$$
$$+\int_{2m}^{p}\int_{0}^{p}\mathbf{1}\left(\frac{x}{2}>c\right)\left(\frac{x}{2}-c\right)d\left(\frac{c}{p}\right)dF(x)+\left[1-F(p)\right]\int_{0}^{p}\mathbf{1}\left(\frac{p}{2}>c\right)d\left(\frac{c}{p}\right)$$

The profit simplifies to:

$$8p\Pi(m)=4\left[F(2m)-F(m)\right]m^2+\int_{2m}^{p}x^2dF(x)+\left[1-F(p)\right]p^2$$
$$\Rightarrow\frac{d\Pi}{dm}>0\Leftrightarrow 2\left[F(2m)-F(m)\right]>mf(m)$$