Income Distribution and the Price Level:

The Balassa-Samuelson Relationship Re-considered*

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Abstract

The Balassa-Samuelson relationship links the per capita income level of a country and its national price level. The fact that these two variables are positively correlated suggests an apparent violation of Purchasing Power Parity (PPP). Balassa and Samuelson offerred a candidate explanation for this finding by appealing to a service component in the national price level. This paper provides an alternative candidate explanation for the B-S relationship, which is based on an appeal to mismeasured quality. No satisfactory method of controlling fully for quality difference in price indexes has yet been developed. Thus, if ricer countries consume higher quality variants of each good, the failure of allowing fully for these quality differences implies the B-S relationship. More interestingly, this candidate explanation leads to a second, distinctive, testable prediction: controlling for a country's GDP per capita, a non-monotonic relationship should exist between the degree of income inequality in the country and its national price level. I show that this second prediction is consistent with empirical evidence.

JEL classifications: E3; D3; O4; F3.

Keywords: price level; distribution; inequality; quality; Balassa-Samuelson.

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1. Introduction

The Balassa-Samuelson relationship, first introduced in 1964[1], links the per capita income level of a country to its price level, as measured by a broad price index. Balassa and Samuelson showed that as we move towards richer countries, the measured price level becomes higher. (the 'Penn effect'.) This represents an apparent violation of Purchasing Power Parity (PPP). Balassa and Samuelson proposed an explanation based on an appeal to the presence of a service component in the national price level. In other words, there are always local costs of processing, distributing etc., which will reflect local wage rates, making the violation of PPP possible, as international arbitrage cannot eliminate the cross-country differentials in local service content.

This classical explanation has been heavily criticized in the literature. For example, Rogoff (1996) showed that there is empirical support for the model when comparisons are made between the set of 'all poor countries' and 'all rich countries', but this effect is not statistically significant within either the poor countries group or the rich countries group (Figure 1).

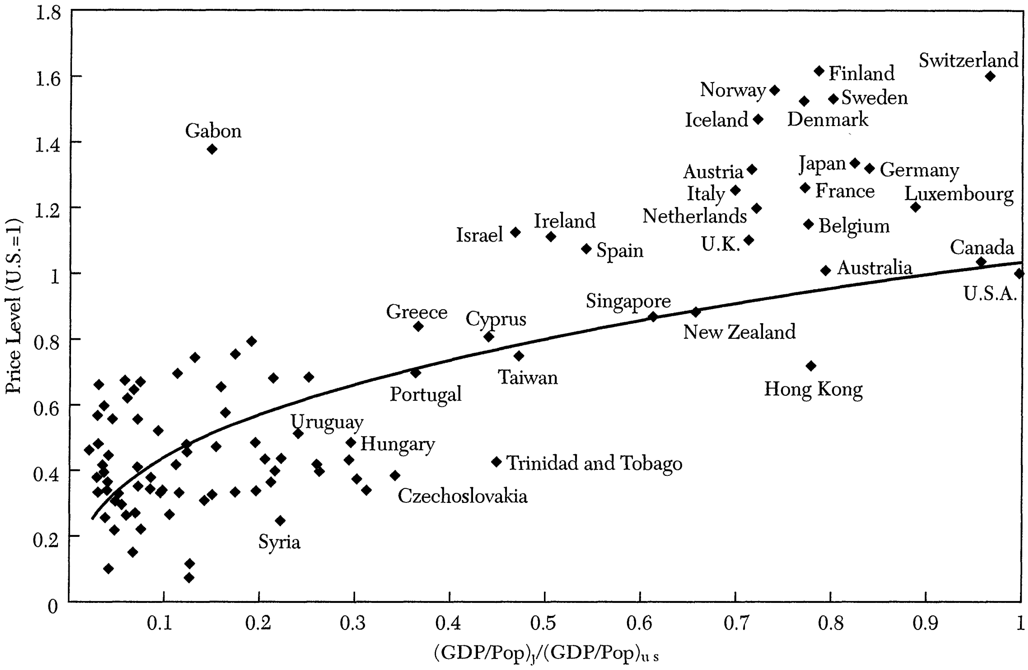
In this paper, an alternative candidate explanation for the B-S relationship is developed based on an appeal to 'mismeasured quality'. The idea that quality is not fully adjusted for in conventional price indices is a long standing theme in the industrial organization literature. The idea can be traced to the hedonic prices literature (Griliches, 1961), and it has been re-explored in recent work by Pakes (2003, 2005). In the present setting, failure to make a full adjustment for quality levels means that, since the average quality level of goods consumed is higher in richer countries, a spurious correlation will appear between a country's income level and its price level.

The model developed in this paper is a hedonic pricing model *à la* Rosen (1974). The key assumption is that income elasticity of quality is positive and is higher for nontraded goods.

To motivate this assumption, note that while households with higher incomes tend

[1] Balassa (1964) and Samuelson (1964)

Figure 1: Price Level versus GDP per capita in 1990 (U.S.=1)



Notes: Source: The Penn World Table 1994

to spend more on all consumption categories[2], different consumption categories (i.e. products) differ in their income elasticities of quantity and quality.[3] For some categories, such as food and housing, as household income rises, the quantity of the good consumed remains constant, but quality rises (Bils and Klenow (2001) and Section 4 below). Thus, the income elasticity of quality is high for these goods, while that of quantity is low. For other categories, such as clothing and footwear, households with higher income tend to consume a larger quantity of the good, while quality remains

[2]The consumption categories are the 2-digit COICOP (Classification of Individual Consumption according to Purpose) divisions, which are Food and non-alcoholic beverages; Alcoholic beverages, tobacco and narcotics; Clothing and footwear; Housing, water, electricity, gas and other fuels; Furnishings, household equipment and routine household maintenance; Health; Transport; Communication; Recreation and culture; Education; Restaurants and hotels; Miscellaneous goods and services.

[3]Here, quantity refers to the number of units consumed by the households, while quality refers to the desirable characteristic within each unit of consumption goods, which is reflected in the unit price. For example, buying the same meal twice doubles the total expenditure on food. Thus, the number of meals is the quantity of food. In contrast, a meal with organic ingredients is more expensive than one with non-organic ingredients. Hence, the ingredients of a meal count as the quality of food. As for housing, two houses with the same characteristics worth twice as much as one, while two identical houses that only differ in their locations have different unit prices. Therefore, the number of houses is quantity and the location of a house is quality. Similarly, the number of clothes is quantity while whether the clothes are of a high street brand or a designer brand is quality.

unchanged. In this case, the income elasticity of quantity is high, relative to that of quality. Those goods with a relatively high income elasticity of quantity, such as clothing and footwear, are usually traded goods, while those with relatively high income elasticities of quality, such as food and housing, are more often nontraded goods. In Section 2 below, I investigate this difference between different categories of goods.

In the theoretical model introduced in the next section, I represent these two categories of goods as follows: households choose one unit of each of a set of nontradable, vertically differentiated goods, offerred at different quality levels, prices being set by reference to a hedonic price function. Household also consume a set of tradable homogeneous goods, the price of which is constant across countries.

Within this model, we examine the relation between a country's income level, and a price index that is not quality adjusted. Given that consumers in the countries with higher per capita income tend to spend more on the nontraded ('quality') goods, this will imply a higher measured price level in richer countries (the B-S or Penn effect). While this first result is intuitively obvious, the model also leads to a less obvious, and distinctive, prediction: controlling for per capita income, income inequality has a positive impact on the national price level within countries with lower per capita income. In other words, the effect of income inequality on the national price level is decreasing in per capita income.

The intuition for this result rests on the idea that income inequality affects the price level of nontraded goods by changing the expenditure share of these goods. The price-quality relationship for vertically differentiated goods is nonlinear and is jointly determined by the distribution of consumers' attributes and cost function parameters. Keeping per capita income constant, a higher degree of income inequality implies a more convex price function of nontraded goods. If the elasticity of substitution between traded goods and nontraded goods is high, then this will lead to a smaller expenditure share for nontraded goods. Since in practice, the quality of nontraded goods cannot be perfectly controlled for, the lower expenditure share on nontraded goods will imply a lower price level for nontraded goods. The Laspeyres index, the country's average price ratio of traded goods and nontraded goods, relative to the U.S., weighted by the expenditure shares of the U.S.. (For the use of the Laspeyres and Paasche indices in this setting, see footnote[4].) This index will now be lower, since the price ratio of non-traded good is lower and there is no change in the price ratio of traded good, or in the weights. Thus, income inequality has a negative impact on the Laspeyres index. However, the impact of income inequality on the Paasche index, which is the average price ratios of traded good and nontraded goods relative to the U.S. weighted by the country's expenditure shares, will depend on the country's per capita income relative to the US. With a low enough per capita income, the price ratio of nontraded good relative to the U.S. will be lower than the price ratio of traded good relative to the U.S., which is always equal to 1, so a lower expenditure share on nontraded goods will increase the Paasche index. With a high per capita income, the relative price ratio of nontraded goods will be higher than the relative price ratio of traded good, hence the lower expenditure share on nontraded goods will reduce the Paasche index. Given that the geometric mean of these two indices is used as a proxy for the national price level, with a low enough per capita income, income inequality will have a positive impact on the national price level, while with a high per capita income, the impact is negative. Therefore, per capita income affects the national price level by changing the price level of the nontraded goods, while income inequality affects the national price level mainly through its impact on the expenditure share of nontraded goods.

In testing the empirical predictions of the theory in what follows, we note that many of the goods (expenditure categories) used in constructing the Penn World Table are

[4]It may be helpful to note how the national price level is constructed in the standard data set used in this literature, i.e. the Penn World Table. To construct the national price level for a country, say the UK, we construct the UK's bilateral Laspeyres and Paasche price index relative to the U.S. The Laspeyres index and the Paasche index are weighted averages of the price ratios of traded goods and nontraded goods of the UK relative to the US. The weights in the Laspeyres index are given by the expenditure shares of the base country, the US, while the weights in the Paasche index are given by the expenditure share of the UK. The two indices are used to construct the national price level of the UK using the Geary-Khamis (GK) method as shown in Deaton and Heston (2010). Deaton and Heston also show that the national price levels in the Penn World Table can be well approximated by the geometric mean of the bilateral Laspeyres index and the Paasche index. In the model of the present paper we use the geometric mean of the two indices as the country's national price level, to mimic its national price level in the Penn World Table.

pure services. (The original B-S explanation rested implicitly in the notion that (all) expenditure categories were tradable manufactured goods that had some local service component). With this in mind, we carry out the empirical test using the standard data, i.e. using an index based on all expenditure categories. We then repeat the exercise using a modified price level index while excludes the 'pure services' categories. It is shown that even when services are excluded, the B-S relationship survives, and the second distinctive prediction of the present theory is consistent with the evidence.

The paper is structured as follows: Sections 2 and 3 develop the model, and examine how income distribution affects price levels and expenditure shares. Section 4 uses detailed consumption expenditure data to justify the main assumption used in the model. In Section 5, the predictions are tested at various levels of aggregation. Section 6 concludes.

2. A 'Mismeasured Quality' Interpretation

2.1. *The Basic Intuition*

The model developed in this section is a hedonic pricing model *à la* Rosen (1974), in which consumers and firms choose their optimal positions along an equilibrium price schedule $p(z)$, where z is a vector of characteristics of the product in question.

The focus of the analysis lies in establishing a relationship between a country's level of income, and – more importantly – the form of income distribution in the country, and the pattern of demand for both 'quality' goods and 'commodity' goods.

The novel prediction of the model is that controlling for per capita income, inequality is correlated with the national price level. The basic intuition is: suppose a country with per capita income μ^*, whose income distribution is made up of three income groups with equal population. The top income group consumes a top quality product, the middle group consumes a middle quality product and the bottom income group consumes a bottom quality product. Given the same Cobb-Douglas utility function, every one spends a same fraction θ of his/her income on the quality products. The implied price of the product will be given by the average expenditure on it, which is equal to $\mu^*\theta$. Now consider a mean-preserving spread of income distribution, which means that there are now less people in the middle income group and more people in the top and bottom groups. This income redistribution will increase the quantity demanded of the top and bottom quality products. As the total cost function of the quality product is convex, this effect will increase the prices of the top and bottom quality products, which will result in a more convex price function. With a more convex price function for the quality product and the unit elasticity of substitution in the Cobb-Douglas utility function, all individuals will respond in this new situation by spending a smaller fraction θ' ($< \theta$) of income on the quality product. As a result, its price level, i.e. the average expenditure on the quality product, is now equal to $\mu^*\theta'$, which is less than before. This is the mechanism through which income inequality affects the measured national price level in the present model.

2.2. *The Model I: The Consumer's Problem*

There is a unit mass of consumers indexed by individual income level c. The income distribution is assumed (conventionally) to follow a Pareto distribution characterized by two parameters k_c and c_m, where c_m is the lower bound of c and k_c is the shape parameter. Hence the probability density function of income is $f(c) = k_c \frac{c_m^{k_c}}{c^{k_c+1}}, k_c > 0, c \in [c_m, \infty)$.

If we decompose the income elasticity of consumption expenditure into an income elasticity of quality and an income elasticity of quantity, it will be shown empirically in what follows that goods differ substantially in their income elasticities of quality and quantity. We will divide goods into three types based on the magnitudes of these two elasticities. The first type of goods, which we call x goods, have zero income elasticity of quality and a non-zero income elasticity of quantity. The second type, which we call z goods, have zero income elasticity of quantity and a non-zero income elasticity of quality. For the third type, both elasticities are non-zero. The fact that some of these elasticities are (close to) zero simply reflects the physical nature of the goods, and so we incorporate these features as given parameters of the model which follows. We begin with a setting where there are just two types of goods, x goods and z goods.[5] We begin from the idea that the x goods, which we may think of as simple 'commodities', are traded internationally at a single price. In other words, we assume purchasing power parity holds for these goods. We simplify notation by choosing the x goods as a numeraire and normalizing their prices to be 1.

We assume the consumer purchases exactly 1 unit of the quality good. Subject to this, consumer preferences are given by a standard Cobb-Douglas utility function $v(x,z) = x^\alpha z^\beta$, $\alpha + \beta = 1$, where z is the quality level of the quality good consumed. Maximizing utility subject to the budget constraint $c = x + p(z)$ yields the consumer's problem

$$\max_{x,z} v(x,z) = x^\alpha z^\beta$$

$$s.t.\ c = x + p(z)$$

Given the homogeneous feature of x goods and its normalized price, the total expenditure on x goods is given by the product of quantity consumed x and its unit price 1. The rest of consumption expenditure will be spent on z goods, which is assumed to be a nonlinear function of quality z.

The Lagrangian is given by $L = x^\alpha z^\beta + \lambda[c - x - p(z)]$. First-order necessary conditions imply that

$$\frac{v_z}{v_x} = p'(z) \text{ and hence } \frac{\beta}{\alpha}\frac{x}{z} = p'(z)$$

Since $x = c - p(z)$, we have $c = \dfrac{\alpha}{\beta} z p'(z) + p(z)$, which implies that if a consumer chooses a vertically differentiated good with quality z, then his/her income must be equal to $\dfrac{\alpha}{\beta} z p'(z) + p(z)$. We denote the consumer's income conditional on choosing quality z by $h(z)$. Recalled that $f(c)$ denotes the pdf of income c, and that $c = h(z)$ whence $z = h^{-1}(c)$. From this we can write down the pdf of z, which we denote as

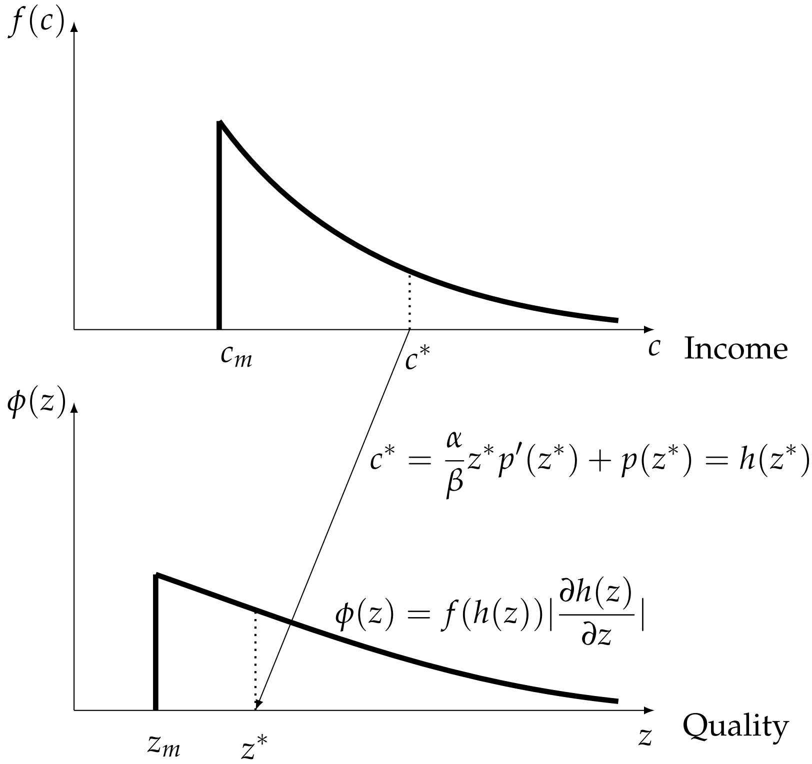
[5] The third type of good, which has nonzero income elasticities of both quality and quantity, can be thought of as a combination of two components, an x component and a z component.

$\phi(z)$, as follows:

$$\phi(z) = f(h(z)) \left| \frac{\partial h(z)}{\partial z} \right| \tag{1}$$

where $|\cdot|$ denotes the absolute value and is used to ensure that $\phi(z)$ is always positive even if $\partial h(z)/\partial z < 0$. The mapping from the pdf of c to the pdf of z can be illustrated in Figure 2.

Figure 2: Mapping From Income Distribution to Distribution of Quality Demanded



Notes: Given the exogenous income distribution and the relationship between income and quality derived from the utility maximization problem, the distribution of quality demanded can be obtained.

Substituting for $h(z)$ and $f(\cdot)$ in (1) yields

$$\begin{aligned} \phi(z) &= f(\frac{\alpha}{\beta} z p'(z) + p(z)) \left| \frac{\partial [\frac{\alpha}{\beta} z p'(z) + p(z)]}{\partial z} \right| \\ &= k_c c_m^{k_c} [\frac{\alpha}{\beta} z p'(z) + p(z)]^{-(k_c+1)} \left| \frac{\alpha}{\beta} [p'(z) + z p''(z)] + p'(z) \right| \end{aligned}$$

If we denote the quantity demanded for the good with quality z by $Q^d(z)$, then the market demand in a small interval dz near quality z is given by the product of the pdf of quality around z, $\phi(z)$, and the length of the interval:

$$Q^d(z)dz = k_c c_m^{k_c}[\frac{\alpha}{\beta}zp'(z) + p(z)]^{-(k_c+1)} \left|\frac{\alpha}{\beta}[p'(z) + zp''(z)] + p'(z)\right| dz$$

2.3. *The Model II: The Producer's Problem*

On the supply side, there is a unit mass of firms producing vertically differentiated goods indexed by their product quality z. The distribution of the firms is assumed to be the Pareto distribution characterized by two parameters k_z and z_m, where z_m is the lower bound of z and k_z is the shape parameter.[6] The pdf of z is assumed to take the form:

$$g(z) = k_z \frac{z_m^{k_z}}{z^{k_z+1}}, k_z > 0, z \in [z_m, \infty)$$

Producers in all countries are assumed to have the same cost function $\Delta(M,z) = A_z M^\tau z^\gamma$, $\tau > 1, \gamma > 1$, where A_z is the productivity parameter and M denotes the number of units of the product with quality z that the firm produces. We assume $\tau > 1$ and $\gamma > 1$, which ensures that total cost is a convex function in M and z.

The producers are price takers. Furthermore, it is assumed that the producers can vary M but not z. (i.e. a producer's quality is a given parameter in the short run). Therefore, the producer's problem is to maximize profit by choosing its output level M of the quality good:

$$\max_M Mp(z) - \Delta(M,z)$$

The first-order conditions imply that

$$p(z) = \frac{\partial \Delta}{\partial M} = A_z \tau M^{\tau-1} z^\gamma$$

$$\text{Thus, } M(z) = (\frac{p(z)}{A_z \tau z^\gamma})^{\frac{1}{\tau-1}}$$

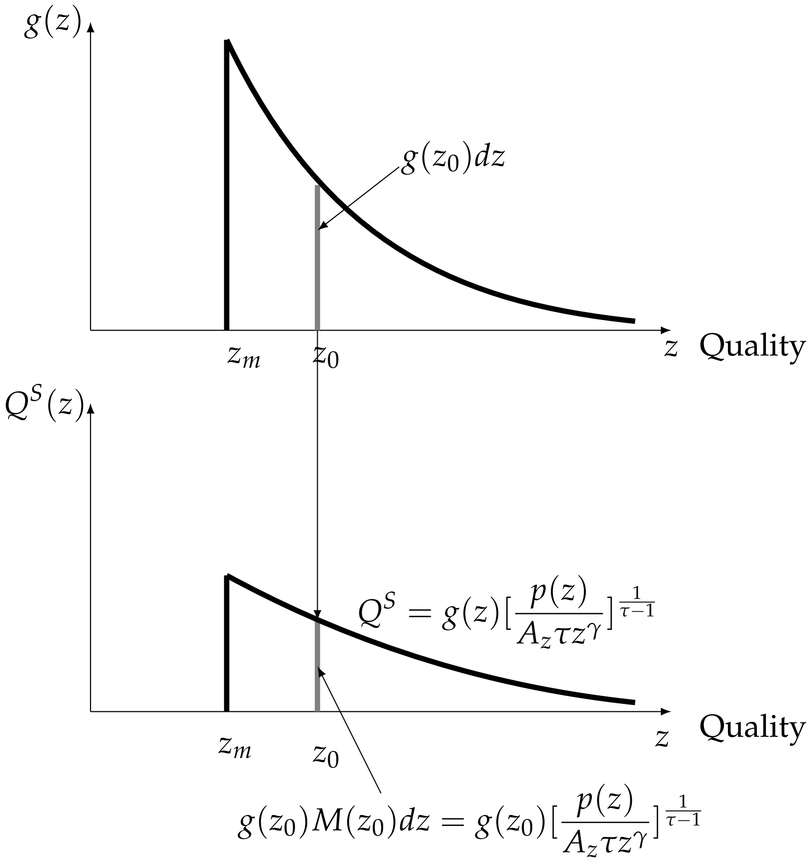
[6] The reasons for using the Pareto distribution are not only that we can get a closed form solution but also that this assumption is consistent with empirical evidence. Gaffeo et al. (2003) analyze the average size distribution of a pool of the G7 group firms over the period 1987-2000. They find that the empirical distributions are all consistent with the power law. In our model, the quality of a firm is a power transformation of the size of the firm, so it is reasonable to assume quality also follows the Pareto distribution as the Pareto distribution is close under power transformation.

If we denote the firm's output of the quality good as $Q^s(z)$, then the market supply in a small interval dz near quality z is given by the product of the pdf of firms around z, the quantity supplied by each z firm and the length of the interval:

$$Q^s(z)dz = g(z)M(z)dz$$
$$Q^s(z)dz = k_z \frac{z_m^{k_z}}{z^{k_z+1}} \left(\frac{p(z)}{A_z \tau z^{\gamma}}\right)^{\frac{1}{\tau-1}} dz$$

Figure 3 shows the mapping from the firm distribution to the distribution of quality supplied.

Figure 3: Mapping From Firm Distribution to Distribution of Quality Supplied



Notes: Given the initial firm distribution and the output of each firm, the distribution of quality supplied can be obtained.

2.4. *Market Equilibrium*

An equilibrium is defined as a triple $\{z(c), M(z), p(z)\}$, where $z(c)$ is the policy functions for consumers, and $M(z)$ for producers, and the price schedule $p(z)$ such that:

1. $z(c)$ solves the consumer's utility maximization problem taking $p(z)$ as given.
2. $M(z)$ solves the producer's profit maximization problem taking $p(z)$ as given.
3. Market clears: demand is equal to supply for z goods, i.e., $Q^s(z) = Q^d(z)$ for all z.

2.5. *An Analytical Solution*

In equilibrium, we must have the market clearing condition $Q^s(z)dz = Q^d(z)dz$. Therefore,

$$\begin{aligned} & k_z \frac{z_m^{k_z}}{z^{k_z+1}} [\frac{p(z)}{A_z \tau z^{\gamma}}]^{\frac{1}{\tau-1}} dz \\ = & k_c c_m^{k_c} [\frac{\alpha}{\beta} z p'(z) + p(z)]^{-(k_c+1)} \left| \frac{\alpha}{\beta}[p'(z) + z p''(z)] + p'(z) \right| dz \end{aligned}$$

$$\begin{aligned} & k_z z_m^{k_z} (\frac{1}{A_z \tau})^{\frac{1}{\tau-1}} z^{-(k_z+1)-\gamma\frac{1}{\tau-1}} p(z)^{\frac{1}{\tau-1}} \\ = & k_c c_m^{k_c} [\frac{\alpha}{\beta} z p'(z) + p(z)]^{-(k_c+1)} \left| \frac{\alpha}{\beta}[p'(z) + z p''(z)] + p'(z) \right| \end{aligned} \tag{2}$$

This is a second-order nonlinear non-autonomous differential equation defining $p(z), z \in [\overline{z}_m, \infty)$. The boundary condition is:

$$c_m = \frac{\alpha}{\beta} \overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m)$$

Here $\overline{z}_m$ is the lowest quality that is viable in the equilibrium, which is determined by the lowest income c_m and the equilibrium price function $p(z)$.

There is no general procedure to obtain the solution of this class of differential equations, so we adopt the standard method of undetermined coefficients, to find a particular solution. We postulate the equilibrium price function is of the form $p(z) = bz^d$, with $d > 0$. We then substitute this form of solution into the market clearing condition to solve for the values of the parameters b and d.

The first and second derivatives of the postulated price function form are

$$p'(z) = bdz^{d-1}, p''(z) = bd(d-1)z^{d-2} \tag{3}$$

Substituting (3) into (2) yields

$$\begin{aligned} & k_z z_m^{k_z} (\frac{b}{A_z \tau})^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} \\ = & k_c c_m^{k_c} [b(\frac{\alpha}{\beta} d + 1)]^{-(k_c+1)} (\frac{\alpha}{\beta} d^2 + d) b z^{-d(k_c+1)+d-1} \end{aligned} \tag{4}$$

Since (4) holds for all z and both LHS and RHS are power functions of z, it must be true that the two parameters of the power functions on both sides are equal

$$k_z z_m^{k_z} (\frac{b}{A_z \tau})^{\frac{1}{\tau-1}} = k_c c_m^{k_c} [b(\frac{\alpha}{\beta} d + 1)]^{-(k_c+1)} (\frac{\alpha}{\beta} d^2 + d) b \tag{5}$$

$$-(k_z+1) + (d-\gamma)\frac{1}{\tau-1} = -d(k_c+1) + d - 1 \tag{6}$$

From (6):

$$d = \frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}} \tag{7}$$

Given equation (7), the expenditure share on z goods can be obtained by

$$\frac{p(z)}{c} = \frac{p(z)}{\frac{\alpha}{\beta} z p'(z) + p(z)} = \frac{bz^d}{\frac{\alpha}{\beta} bdz^d + bz^d} = \frac{1}{\frac{\alpha}{\beta} d + 1} \tag{8}$$

From (7), (8) and the fact that $Gini = \dfrac{1}{2k_c - 1}$ for the Pareto distribution, we can derive how the Gini coefficient affects the convexity of the price function and hence the expenditure share, which is stated in Proposition 1.

Proposition 1 *(Income Distribution and Expenditure Share) Income inequality has a positive impact on the expenditure share of x goods and a negative impact on the expenditure share of z goods. Per capita income has no impact on expenditure shares.*

The intuition behind the proposition is that an increase in the Gini coefficient implies an increase in d and hence a more convex price function. Since the price of x is 1 and z has a non-linear price function, an increase in the convexity of the price function of z will make people spend less fraction of their expenditure on z and more on x due to the high substitutability between the two goods. This mechanism about how income inequality affects expenditure share is crucial in determining how income inequality influences the price level, which will be provided in the next section.

Substituting (7) into (5) can solve for the other parameter b in the price function:

$$b = \left\{ \frac{k_c c_m^{k_c} \left[\frac{\alpha}{\beta} \frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}} + 1 \right]^{-k_c} \frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}}}{k_z z_m^{k_z} \left(\frac{1}{A_z \tau}\right)^{\frac{1}{\tau-1}}} \right\}^{\frac{1}{k_c + \frac{1}{\tau-1}}} \tag{9}$$

Therefore, one solution to the differential equation is

$$p(z) = bz^d = \left\{ \frac{k_c c_m^{k_c} \left[\frac{\alpha}{\beta} \frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}} + 1 \right]^{-k_c} \frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}}}{k_z z_m^{k_z} \left(\frac{1}{A_z \tau}\right)^{\frac{1}{\tau-1}}} \right\}^{\frac{1}{k_c + \frac{1}{\tau-1}}} z^{\frac{k_z + \gamma\left(\frac{1}{\tau-1}\right)}{k_c + \frac{1}{\tau-1}}}, z \in [\overline{z}_m, \infty)$$

where $\overline{z}_m$ satisfies

$$c_m = \frac{\alpha}{\beta} \overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m)$$

After obtaining the equilibrium price function, before aggregating it and analysing how income distribution influences the national price level, we can first investigate how income distribution affects prices at product level, i.e. how the difference in income distribution affects the relative price of a product with a particular quality z_0 between two countries.

Suppose the hedonic price functions in country i and country j are $p_i(z) = b_i z^{d_i}$ and $p_j(z) = b_j z^{d_j}$, where b_i, b_j, d_i and d_j are determined as in the equilibrium price function. Then the price ratio of a product with quality z_0 between the two countries is

$$\frac{p_i(z_0)}{p_j(z_{0)}} = \frac{b_i}{b_j} z_0^{(d_i - d_j)}$$

If we keep the income distribution of country j constant, and increase the per capita income of country i while keeping its Gini coefficient constant, this will imply an increase in b_i and hence an increase in the price ratio for all values of z_0. If we keep the income distribution of country j constant, and increase the Gini coefficient of country i while keeping its per capita income coefficient constant, this will imply a decrease in b_i and an increase in d_i. The increase in d_i will imply a higher convexity of the price ratio function. When there are changes in both per capita income and the Gini coefficient, the direction of the change in b will depend on the values of the parameters but the positive relationship between d and the Gini coefficient still holds.

The above results can be formalized as follows:

Proposition 2 *If the price ratio of the same good between two countries i and j* $\frac{p_i(z_0)}{p_j(z_0)}$ *is a function of the quality of that good* z_0*, then the difference in income inequality between the two countries determines the power of the price ratio function. Specifically, if the Gini coefficient of country i is higher (lower) than that of country j, then the price ratio function is upward (downward) sloping.*

We now explore the implications of Proposition 1 and 2 for the B-S relationship, in two alternative settings:

1. Perfect quality measurement.

2. A setting where quality is not measured as in Pakes (2003).

The B-S relationship is a relationship between a country's level of income and its national price level of final goods. As consumption bundles consist of the x goods and the z goods in our model, to measure the national price level, ideally we want to use observed data to reveal the unit price of the x goods and the price schedule of the z goods $p(z)$. Then the price schedule $p(z)$ can be used to construct a price index of the z goods. Finally, the unit price of the x goods and the price index of the z goods are aggregated into a national price level. The above procedures of measurement and aggregation face, however, a prominent practical issue. The issue is that quality cannot be perfectly controlled for the z goods. Pakes (2003) shows how to use hedonics to ad-just quality biases in the price indexes of quality goods due to the introduction of new goods. The adjustment procedures require a complete dataset on the characteristics of the goods, which is impossible in reality. Without the level of quality being observed, the observed prices of the z goods cannot tell us anything about the price level of the z goods, as their prices depend on both the level of quality z and the parameters b and d in the price function. The measurement issue at the data collecting stage will also contaminate the aggregation procedure. Without knowing the underlying price schedule $p(z)$, the common practice of constructing the price index of the z goods is to use the simple average of the observed prices as its price index. As a result, a higher price index of the z goods could be either due to higher values of b and d in the price function or simply due to the fact that the prices of higher quality goods have been observed.

Suppose quality can be properly measured, which means that we are able to reveal the underlying price function $p(z)$, then equation (9) implies that there still exists the B-S relationship. To see this, suppose we keep a country's Gini index constant and increase its per capita income, this implies a constant k_c but a higher c_m in the Pareto distribution. From (7) and (9), d will stay constant as before but b will go up, resulting in an upward shift of the price schedule $p(z)$. Hence, for any quality goods, the price will be higher than before. This is because a higher level of per capita income will increase the demand for the higher quality goods. The resulting higher output will increase their prices as the marginal cost is increasing in output. The elasticity of b with respect to c_m is equal to $\frac{k_c}{k_c + \frac{1}{\tau-1}}$. However, given reasonable values of the parameters, the elasticity is quantitatively small.

If quality cannot be adjusted as in Pakes (2003), we have to use the simple average of the observed prices of the z goods as its price index. Suppose we again keep a country's Gini index constant and increase its per capita income, now the price index of the z goods not only reflects a higher b as in the previous situation but also reflects the fact that now the country will consume goods with higher qualities than before. This will cause an upward bias in the price index of the z goods and hence in the national price level. Moreover, as income inequality can affect the convexity of the price function and the expenditure share on z goods as shown in Proposition 1 and 2, it will have an impact on the price index of the z goods and the national price level. The details are shown below.

2.6. *The Aggregate Price Level*

Given the market equilibrium price schedule $p(z)$, we can calculate the average price level of z goods $\overline{p}$, which is the total expenditure on z goods divided by the total number of units.

$$\begin{aligned}
\overline{p} &= \frac{\int_{\overline{z}_m}^{\infty} p(z) Q^s(z) dz}{\int_{\overline{z}_m}^{\infty} Q^s(z) dz} \\
&= \frac{\int_{\overline{z}_m}^{\infty} bz^d k_z z_m^{k_z} (\frac{b}{A_z \tau})^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz}{\int_{\overline{z}_m}^{\infty} k_z z_m^{k_z} (\frac{b}{A_z \tau})^{\frac{1}{\tau-1}} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz} \\
&= b \frac{\int_{\overline{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d} dz}{\int_{\overline{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})} dz} \\
&= b \frac{\frac{z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1}}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1} \Big|_{\overline{z}_m}^{\infty}}{\frac{z^{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1} \Big|_{\overline{z}_m}^{\infty}}
\end{aligned}$$

If we assume $-(k_z+1)+(d-\gamma)(\dfrac{1}{\tau-1})+d+1 < 0$, then

$$\overline{p} = \frac{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1} b\overline{z}_m{}^d$$

Since $c_m = \dfrac{\alpha}{\beta}\overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m)$ and $p(z) = bz^d$, $b\overline{z}_m^d = \dfrac{\beta}{\alpha d + \beta} c_m$.
Therefore, we have

$$\overline{p} = \frac{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+1}{-(k_z+1)+(d-\gamma)(\frac{1}{\tau-1})+d+1} \frac{\beta}{\alpha d + \beta} c_m \tag{10}$$

Substituting (7) into (10) yields

$$\overline{p} = \frac{k_c}{k_c - 1} c_m \frac{\beta}{\alpha \frac{k_z + \gamma(\frac{1}{\tau-1})}{k_c + \frac{1}{\tau-1}} + \beta}$$

As the Gini coefficient and the mean of the Pareto income distribution are equal to

$\frac{1}{2k_c - 1}$ and $\frac{k_c c_m}{k_c - 1}$, we can express the average price level in terms of the mean and the Gini coefficient

$$\overline{p} = \mu \frac{\beta}{\alpha \frac{k_z + \gamma\left(\frac{1}{\tau - 1}\right)}{\frac{\frac{1}{Gini}+1}{2} + \frac{1}{\tau-1}} + \beta} \tag{11}$$

where $\mu = \frac{k_c}{k_c - 1} c_m$ and $Gini$ are the mean and the Gini coefficient of the income distribution. This equation tells us the effects of per capita income and income inequality on the average price level of z goods, which is summarized in Proposition 3.

Proposition 3 *(Income Distribution and the Disaggregate Price Level) Per capita income has a positive impact on the average price level of z goods, whereas income inequality has a negative impact. Therefore, the elasticity of the average price of z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative.*

Proof: See Appendix A.1.

Equation (11) also shows that the effect of per capita income on the average price level of z goods depends on income inequality and the effect of income inequality depends on per capita income.

Proposition 4 *The effect of income inequality on the average price level of z goods (the absolute value of* $\frac{\partial \overline{p}}{\partial Gini}$*) is increasing in per capita income* μ*, while the effect of per capita income on the average price level of z goods (*$\frac{\partial \overline{p}}{\partial \mu}$*) is decreasing in income inequality.*

Proof: See Appendix A.2.

To investigate the implications of income distribution for the national price level, we need to construct an aggregate price index.

Although the vertically differentiated goods are produced by local firms, the homogeneous goods are tradable goods with their price level equalized across countries. Therefore, cross-country price comparison is still meaningful as we can compare the national price level using the price level of the homogeneous goods as an anchor or a numeraire.

Here for simplicity and in order to derive analytical results, we define the aggregate price level as the average price of x goods and z goods weighted by expenditure shares as in the Laspeyres or Paasche index. In Proposition 5, the results regarding how income distribution affects the aggregate price level are shown.

To make the results comparable with the empirical evidence, the results regarding how income distribution affects the log of the aggregate price level are also shown.

Proposition 5

(a) Income Distribution and the Paasche Index: If we define the aggregate price as the Paasche index

$$P_P = \frac{1}{1} share_x + \frac{\overline{p}}{\overline{p}_0} share_z = 1 \frac{\alpha d}{\alpha d + \beta} + \frac{\overline{p}}{\overline{p}_0} \frac{\beta}{\alpha d + \beta},$$

where zero is used in the subscript to denote the variables from the base country, i.e. the U.S., then per capita income has a positive impact on the aggregate price level, or the elasticity of the aggregate price level with respect to per capita income ($e_{P_P,\mu} \equiv \frac{\partial P_P}{\partial \mu} \frac{\mu}{P_P}$) is positive. Moreover, the impact of income inequality on the aggregate price level and the semi-elasticity of the aggregate price level with respect to income inequality ($e_{P_P,Gini} \equiv \frac{\partial P_P}{\partial Gini} \frac{1}{P_P}$) depend on the per capita income relative to the U.S.. They are both positive when per capita income is low enough relative to the U.S. while they are both negative when per capita income is high.

(b) Income Distribution and the Laspeyres Index: If we define the aggregate price as the Laspeyres index

$$P_L = \frac{1}{1} share_{x,0} + \frac{\overline{p}}{\overline{p}_0} share_{z,0} = 1 \frac{\alpha d_0}{\alpha d_0 + \beta} + \frac{\overline{p}}{\overline{p}_0} \frac{\beta}{\alpha d_0 + \beta}.$$

Then per capita income has a positive impact on the aggregate price level, i.e. the elasticity of the aggregate price level with respect to per capita income ($e_{P_L,\mu} \equiv \frac{\partial P_L}{\partial \mu} \frac{\mu}{P_L}$) is positive, whereas income inequality has a negative impact, or the semi-elasticity of the aggregate price level with respect to income inequality ($e_{P_L,Gini} \equiv \frac{\partial P_L}{\partial Gini} \frac{1}{P_L}$) is negative.

(c) No matter whether the aggregate price level is defined as the Laspeyres index or the Paasche index, the effect of per capita income on the aggregate price level ($\frac{\partial P_P}{\partial \mu}$ and $\frac{\partial P_L}{\partial \mu}$) is decreasing in income inequality and the effect of income inequality ($\frac{\partial P_P}{\partial Gini}$ and $\frac{\partial P_L}{\partial Gini}$) is decreas-

ing in per capita income μ. Moreover, the elasticity of the aggregate price level with respect to per capita income $e_{P_P,\mu}$ and $e_{P_L,\mu}$ is decreasing in income inequality whereas the semi-elasticity of the aggregate price level with respect to income inequality $e_{P_P,Gini}$ and $e_{P_L,Gini}$ is decreasing in per capita income μ.

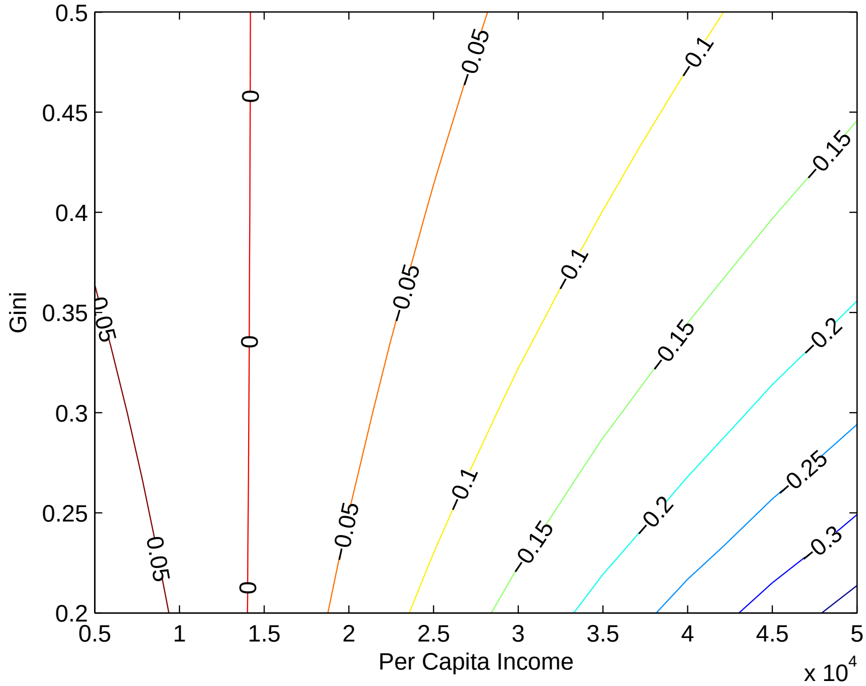
$Proof$: See Appendix A.3.

In practice, the way to construct the multilateral price index as in the International Comparison Program (ICP) is different. However, as shown in Deaton and Heston (2010), it can be approximated very well by the bilateral Fisher index, i.e., a geometric mean of the Laspeyres and the Paasche index. Therefore, the results in Proposition 5 can be used to show how income distribution affects the bilateral Fisher index or the national price level.

No matter whether the aggregate index is defined as the Laspeyres index or the Paasche index, the elasticity of the national price level with respect to per capita income is always positive and it is decreasing in income inequality. Since the elasticity of the Laspeyres index with respect to income inequality is negative and the elasticity of the Paasche index with respect to income inequality is decreasing in per capita income, with a low enough per capita income, the elasticity of the bilateral Fisher index is positive while it is negative with a high per capita income. This is confirmed in Figure 4, where the derivatives of the bilateral Fisher index with respect to income inequality $\frac{\partial P_F}{\partial Gini}$ for different combinations of per capita income and income inequality are plotted. For a lower level of per capita income $\frac{\partial P_F}{\partial Gini}$ is positive, while it is negative when per capita income is high.

The intuition behind the proposition is: with higher per capita income, a country will spend a larger amount of its income on z goods, which implies a higher average price of z goods due to the imperfect control over quality. Therefore, with a constant price of x goods, higher per capita income implies a higher aggregate price level. However, with a higher Gini coefficient, a country will spend a smaller fraction of income on z goods. Since in practice, the quality of z goods cannot be easily controlled, the lower

Figure 4: The Contour of the Effect of Income Inequality on the Bilateral Fisher Index ($\frac{\partial P_F}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality.



Notes: The base country income distribution is calibrated using U.S. data in 2005.

expenditure share on z goods will imply a lower measured price level for z goods. The Laspeyres index, which is the average price of x and z relative to the U.S. weighted by the expenditure shares of the U.S., will be lower, since the relative price of z is lower and there is no change in the relative price of x and the weights. However, the impact on the Paasche index, which is the average price of x and z relative to the U.S. weighted by the country's expenditure shares, will depend on the country's per capita income relative to the United States. With a low enough per capita income, the price of x relative to the U.S. will be higher than the price of z relative to the U.S.. So a lower expenditure share on z goods will increase the measured aggregate price level. With a higher per capita income, the relative price of z goods will be comparable with or higher than the relative price of x goods, so the lower expenditure share on z goods will reduce the aggregate price level. Since the product of the expenditure share and the average price of z enters the aggregate price level, a higher per capita income, which increases the average price of z, will strengthen the effect of income inequality, while a lower income inequality, which increases the expenditure share of z, strengthens the effect of per capita income. Hence the effect of per capita income must be decreasing in income inequality and the effect of income inequality must be decreasing in per capita income.

Next, we ask: what is the key feature of the present model that leads to Proposition 5. To address this question, we compare our model with the classic model of vertical product differentiation.

As quality is not controlled for in the price index of quality products, the price index of quality products is measured as the average expenditure on quality products. Keeping per capita income constant, the price index of quality products will only depend on its expenditure share. Therefore, whether income inequality can affect the price index of quality products crucially depends on whether income inequality can affect the expenditure shares.

In both the classic model of vertical product differentiation and the model in Section 2, the expenditure share of the quality products crucially depends on the convexity of the price schedule of quality products. In the former model, the price schedule is usually exogenously given, so the expenditure share is constant and cannot be affected by income inequality. This closes off the relation between the price level and income inequality. In the latter model, the price schedule is endogenously determined by the firm distribution and income distribution. As a result, income inequality can affect the convexity of the price schedule and hence the expenditure share. Therefore, the price index of quality products will be affected by income inequality. Since the national price level is a weighted average of the price levels of the quality products and commodity goods, the national price level will also be affected by income inequality.

3. Empirical Evidence I: Income Distribution and the Aggregate Price Level

The model developed in Section 2 predicts the B-S relationship, but it also predicts a new relationship between income inequality and the national price level, which is summarized in Proposition 5 above. The new relationship can be summarized as follows:

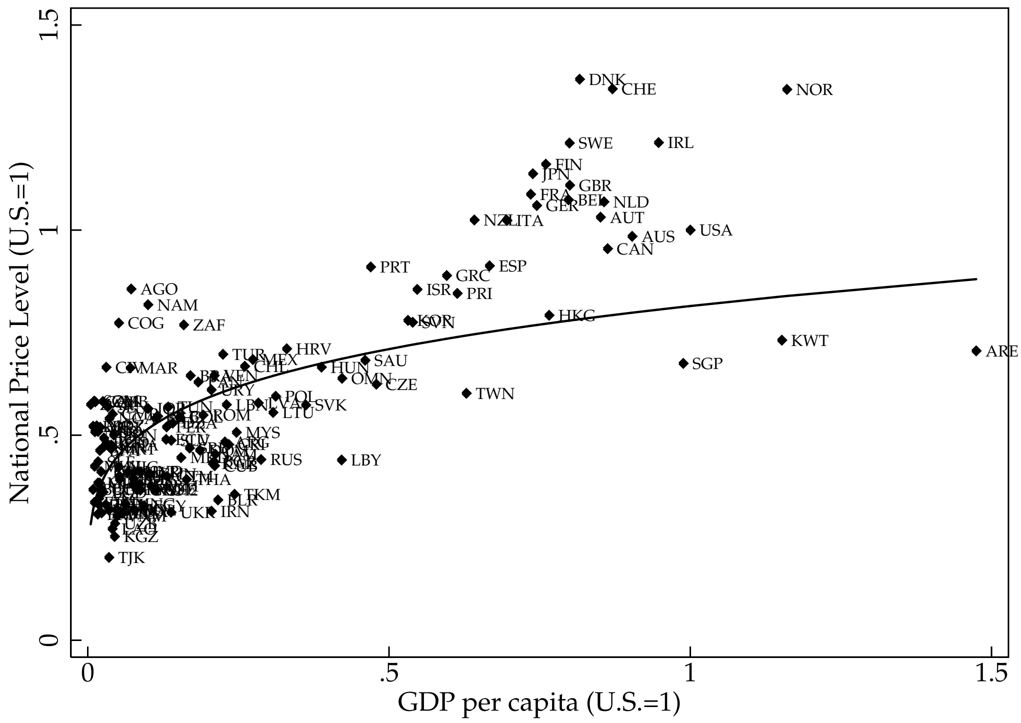
Controlling for per capita income, income inequality is correlated with the national price level: within countries with lower per capita income, income inequality is positively correlated with the national price level, while within countries with higher per capita income, the correlation is negative.

In this section, we investigate this prediction directly using data at the aggregate level. In the next section, we investigate some additional predictions of the model that follow from Proposition 1 and 3 at the disaggregate level.

To show that not only per capita income but also income inequality is important in determining the aggregate price level, we extend the regression in Rogoff (1996) by adding the Gini coefficient as an extra regressor to investigate if the Gini coefficient helps to explain national price differentials.

First, the figure in Rogoff (1996) is reproduced in Figure 5 using 2005 data excluding countries whose population was less than two million in 2005. The data on prices and income are from Penn World Table PWT 7.1. The data on the Gini coefficients are taken from the World Bank: World Development Indicators. Figure 5 shows that the problem with the Balassa-Samuelson hypothesis still persists; it performs well for the whole sample, but does not perform well either within poor countries or within rich countries.

Figure 5: Price Level versus GDP per capita in 2005 (U.S.=1)



Notes: Source: The Penn World Table 7.1

The regressions with the Gini coefficients for the year 2005 are shown in Table 1. Results from Regressions (1) to (5) are consistent with the fact that countries with higher per capita income tend to have higher price levels as the estimated coefficients of relative income are all significantly positive. However, the Gini coefficient in Regression (2) is not significant, while when the product of the Gini index and relative income is included as an interaction term in Regression (3), both the Gini index and the interaction term become significantly negative. This implies a negative relationship between income inequality and the national price level if country *j*'s per capita income is similar to that of the U.S., and this effect is decreasing in per capita income. This also explains why the Gini index in Regression (2) is not significantly negative. This is because if Regression (2) fails to include the interaction term which has significant explanatory power, the estimated coefficient of the Gini index will be the sum of the estimated coefficient of the Gini index in Regression (3) and the product of the estimated coefficient

Table 1: The Effects of the Gini Coefficient in 2005

Independent Variable	Relative Price Level $\log(P_j/P_{U.S.})$ (1)	(2)	(3)	(4)	(5)
$\log(Y_j/Y_{U.S.})$	0.199***	0.214***	0.733***	0.794***	0.720***
	(0.0188)	(0.0213)	(0.105)	(0.111)	(0.106)
Gini Index		0.213	-2.632***	-2.548***	-2.573***
		(0.324)	(0.639)	(0.646)	(0.641)
Gini Index$\times \log(Y_j/Y_{U.S.})$			-1.381***	-1.467***	-1.346***
			(0.275)	(0.291)	(0.277)
VAT				0.00725	
				(0.00497)	
Population					-0.000175
					(0.000164)
Constant	-0.205***	-0.250*	0.773***	0.651**	0.759***
	(0.0472)	(0.128)	(0.235)	(0.270)	(0.235)
Observations	141	128	128	111	128
R^2	0.445	0.458	0.550	0.644	0.554

Note: Data on price and income are taken from the Penn World Tables 7.1. Data on the Gini Index are taken from World Bank: World Development Indicators. VAT data is from PWC "Corporate Taxes, Worldwide Summaries" and International VAT and IPT Service. Population data is from WRDS. ***, ** and * indicate statistically significant different from zero at 1%, 5% and 10% level respectively.

of the interaction term in Regression (3) and the relative income. Since in the sample, most of the relative incomes in logarithm are negative, when they are multiplied with the negative coefficient of the interaction terms, they reduce the magnitude of the negative coefficient of the Gini index and make it insignificant in Regression (2). Therefore, the results show that per capita income has a positive impact on the aggregate price level, i.e. the Penn effect, while income inequality also has a significant impact on the aggregate price level and the impact is decreasing in per capita income. These are consistent with the model predictions in Proposition 5 that per capita income has a positive impact on the national price level and the impact of income inequality on the national price level is decreasing in per capita income. Regressions (4) and (5) control for VAT and population, with the latter being a proxy for market size. The estimation results show that the inclusion of these two control variables does not change the estimation result in (3). Moreover, both control variables are not significant at the 10% level.

Table 2: Different Behaviors of Income Distributions in Different Samples in 2005

	Relative Price Level $\log(P_j/P_{U.S.})$			
	Threshold= 1/3 of $Y_{U.S}$			
	Poor Countries		Rich Countries	
Independent Variable	(1)	(2)	(3)	(4)
$\log(Y_j/Y_{U.S.})$	0.0658**	0.0656***	0.426***	0.521***
	(0.0254)	(0.0247)	(0.127)	(0.114)
Gini Index		1.184***		-1.482**
		(0.294)		(0.560)
Constant	-0.620***	-1.113***	0.0453	0.635***
	(0.0732)	(0.144)	(0.0583)	(0.191)
Observations	106	101	35	27
R^2	0.061	0.197	0.255	0.531

Note: Data on price and income are taken from the Penn World Tables 7.1. Data on the Gini Index are taken from World Bank: World Development Indicators. ***, ** and * indicate statistically significant different from zero at 1%, 5% and 10% level respectively.

Given the significance of the interaction term, to further understand how income distribution affects the national price level in poor and rich countries, in Table 2 the whole sample is split into two subsamples according to the relative income level, and the above regressions are run for the two subsamples. The threshold is set to 1/3 of the per capita GDP of the U.S.. The Penn effect is confirmed in Regressions (1) to (4) as the estimated coefficients are all significantly positive. However, within poor countries, income inequality has a positive impact on the national price level; whereas within rich countries, the impact is negative. This is consistent with the negative coefficient of the interaction term in Table 1, i.e. the impact of income inequality on the national price level is decreasing in per capita income. Moreover, this is also consistent with the model prediction shown in Figure 4 that within countries with lower per capita income, income inequality has a positive impact on the national price level, while within countries with higher per capita income, the impact is negative. In terms of R^2, it can be seen that within poor countries, the inclusion of the Gini coefficient increases the R^2 by 14%, while within rich countries the inclusion of the Gini coefficient increases the R^2 by 27%. These results have confirmed that the relationship between per capita income and national price level is far less impressive both within poor countries and within rich countries. Moreover, income inequality plays an important role in explaining national price differentials, especially within rich countries. As for the quantitative impact of income inequality, the estimated coefficients in Regression (4) imply that aggregate price level increases by about 0.52 percentage points with the 95% confidence interval being $[0.286, 0.757]$ in response to a one percentage point increase in per capita income, whereas the price level decreases by about 1.48 percent with the 95% confidence interval being $[0.327, 2.638]$ in response to a one hundred basis points increase in the Gini coefficient. Moreover, as the Gini coefficients are usually measured with large errors, the magnitude of the estimate is probably biased downwards. The above results are robust to the choice of the threshold. For example, using half of U.S. GDP per capita as the threshold only changes the results quantitatively but not qualitatively.

As have been shown in the model, the positive relationship between per capita income and the national price level in Table 1 is due to the fact that quality is not controlled for. For example, in Bils and Klenow (2001), they use the U.S. data to show that quality growth of 66 durable goods causes an over-estimation of inflation by 2.2%. If quality cannot be controlled for, then it will show up in the price index. Moreover, due to the fact that income elasticity of quality for many consumption goods are non-negligible and tend to be higher for nontraded goods, which are priced in a non-linear way, income distribution will matter for people's choice of quality and will affect the national price level through the price of nontraded goods. This is why income inequality also affects the measured national price level.

4. Empirical Evidence II: Income Distribution, Disaggregate Price levels and Expenditure Shares

In this section, we examine the further predictions of the model that follow from Proposition 1 and 3. For convenience, we repeat the statements of these propositions as follows:

Proposition 1 *(Income Distribution and Expenditure Share) Income inequality has a positive impact on the expenditure share of x goods and a negative impact on the expenditure share of z goods. Per capita income has no impact on expenditure shares.*

Proposition 3 *(Income Distribution and the Disaggregate Price Level) Per capita income has a positive impact on the average price level of z goods, whereas income inequality has a negative impact. Therefore, the elasticity of the average price of z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative.*

These propositions can be tested using a 2-step procedure, as follows:

Step 1: Using consumption expenditure data, we can identify which good in the consumption bundle is more like the x goods and which good is more like the z goods.

Step 2: To test if the empirical effects of income distribution on the price level and expenditure share of the x and z goods are the same as predicted in Proposition 1 and 3.

Since the aggregate price level is an average price level of consumption weighted by expenditure shares, we have to understand the aggregation methods used in practice in order to show that both the assumptions in the model and the model mechanism are consistent with the data. In the construction of both national price indices such as the CPI and multilateral price indices in the Penn World Table, the first step is to construct the sub-indices for different components of consumer expenditure. Then, expenditure data from each country's national account is used to construct the weights for different components and then all the sub-indices are aggregated into an aggregate price index using these weights. However, some aspects of this aggregation method can have important consequences.

As it will be shown in Section 4.1, for consumption goods such as food and housing, the income elasticity of quantity is close to zero, while for consumption goods such as clothes, the income elasticity of quality is close to zero. Based on whether income elasticity of quality is zero or income elasticity of quantity is zero or both are nonzero, we can identify three types of goods. We call the first type x goods and the second type z goods. This observation combined with the aggregation method can have two consequences. Firstly, due to the lack of data on the characteristics of goods, the aggregation method is not able to control quality, hence the higher quality of z goods will be translated into a higher price. Secondly, as has already been shown in the model, income inequality affects the price function and the expenditure share of z goods, and hence the aggregate price level.

Guided by the dichotomy of x and z goods, to understand how income distribution influences the aggregate price level, Section 4.2 investigates how disaggregate prices and expenditure shares change with income distribution, which can be used to show that the mechanism of the model is consistent with the data.

4.1. *Identification of x goods and z goods*

In the traditional literature, prices usually do not play a role and consumption (physical quantity) is equivalent to consumption expenditure given that the price function is linear and unit price is constant. Hence the income elasticity for one good is the income elasticity of consumption (or consumption expenditure) for that good.

However, in the model, because a type z good is priced in a non-linear way, the equivalence between consumption and consumption expenditure is broken.

In general, since expenditure is the product of quantity and unit price, which depends on the quality of the good, income elasticity of expenditure can be decomposed into income elasticity of quantity and income elasticity of quality (unit price) as follows:

$$\begin{aligned}
e_{expenditure} &= \frac{dlog(consumption\ expenditure)}{dlog(income)} \\
&= \frac{dlog(quantity \times unit\ price)}{dlog(income)} \\
&= \frac{d[log(quantity) + log(unit\ price)]}{dlog(income)} \\
&= \frac{dlog(quantity)}{dlog(income)} + \frac{dlog(unit\ price)}{dlog(income)} \\
&= e_{quantity} + e_{quality}
\end{aligned}$$

Moreover, consumption goods differ in their income elasticities of quantity and quality. Here we try to divide all the consumption goods into two groups according to their relative magnitudes of these two elasticities. The first type of goods which we call x goods has very low income elasticity of quality but high income elasticity of quantity. Given this fact, it is assumed that consumers can only change the quantity of x goods but not the quality. The other type of goods which we call z goods has very high income elasticity of quality but low income elasticity of quantity. Similarly, it is assumed that consumers can only change the quality of z goods but not the quantity.

Now, expenditure and quantity data from the U.S. are used to identify which category a particular consumption good belongs to. We focus on four categories of consumption goods, namely food, housing, clothes and vehicles. It is shown below that the income elasticities of quantity of food and housing are close to zero and the income elasticity of quality of clothes is close to zero, while both elasticities are nonzero for vehicles. Table 3 also shows that these four categories plus hotels and restaurants account for on average more than 60% of the total expenditure within OECD countries, and the expenditure on z goods constitutes on average around 70% of the total expenditure of these five categories, hence it is important to incorporate the dichotomy of x goods and z goods into the model due to its significant expenditure share.[7]

[7]Since food at restaurant is a substitute of food at home and staying at a hotel is a substitute of staying at home, given the z goods feature of food and housing, hotels and restaurants should be z goods as well.

Table 3: Expenditure Shares of Food, Housing, Apparel, Transportation and Restaurants and Hotels

Country	Share of the Five Categories	Share of z goods	Share of z goods within the Five Categories
Max	0.751	0.508	0.805
Min	0.466	0.314	0.632
Average	0.606	0.426	0.703
Standard Deviation	0.051	0.043	0.042

Food

As has been documented in the literature, calorie intake does not vary with permanent income across households. Specifically, Aguiar and Hurst (2005) find that employed household heads with a higher income consume similar amounts of calories as employed household heads with a lower income. However, conditional on log calories, they find that the income elasticities of vitamin A and vitamin C are over 0.30 and the income elasticities of vitamin E and calcium are 0.17 and 0.08, respectively. In addition, the income elasticity of cholesterol is negative.

The results suggest that the income elasticity of quantity for food is very close to zero. Households with higher income do not consume a larger quantity of food than households with lower income. Instead, they consume higher quality foods, such as those rich in vitamin and calcium. On the other hand, low income households consume cheaper calories by having a higher composition of fat and cholesterol in their diets.

Housing

As has been defined previously, the number of houses counts as quantity whereas other characteristics of a house, such as square meters, all count as quality. Micro-evidence has shown that the price function of housing is nonlinear. For example, Anderson (1985) estimates the hedonic price function of housing, i.e. regressing the housing price on characteristics of the house which include structural characteristics of the house, improvements to the house, physical characteristics of the lot, neighbourhood characteristics, etc. He shows that the price function is convex. Even if we define housing by square meters, the price function is still estimated as a convex function. For example, Coulson (1992) estimates a nonparametric response of housing price to floorspace size. The marginal price is estimated to be increasing, which implies a convex price function. Mason and Quigley (1996) estimate the hedonic price indices for downtown Los Angeles and they find the price function is convex in size (1000 sq ft). Also, Bao and Wan (2004) find that the sale price per square foot is increasing in gross area controlling for other characteristics using Hong Kong data.

We use data from the SCF (Survey of Consumer Finances) to compute the number of residences, total value of residences and value per residence by percentile of income. Figure 6 plots the results for the year 2004.

By inspection, one can see that as we move from the low percentile of income to the high percentile of income, the number of residences only changes modestly and almost all the variation in total value of residence is due to the variation in value per residence. This implies a very low income elasticity of quantity.

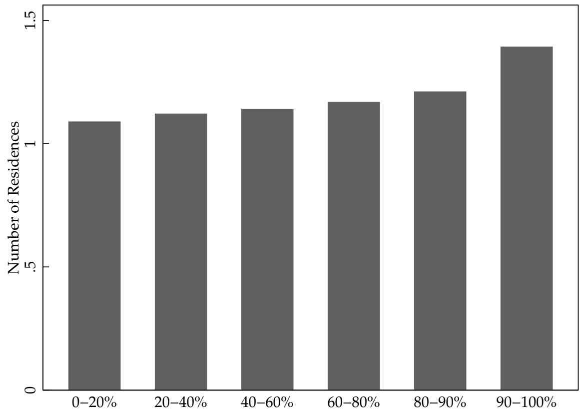
Clothes

The detailed expenditure data on clothes are extracted from the raw data files of the Consumer Expenditure Survey (CEX), which includes both the expenditure on and quantity of clothes. The total expenditure on a certain type of clothes is divided by the number of clothes to get the unit price. Figure 7 plots the total expenditure and the unit price for different types of clothes across the nine income classes in the CEX. The white bars denote the total expenditure and the black bars denote the unit price. It can be seen that for all types of clothes, unit prices are almost the same for all income classes, which suggests a very low income elasticity of quality.

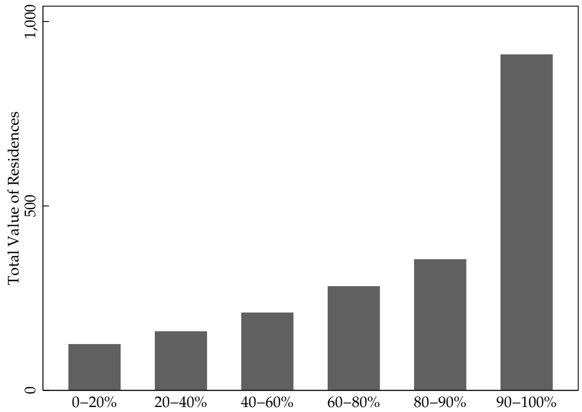
Vehicles

Data on the quantity and expenditure of vehicles is also available from the CEX. As has been done for housing, Figure 8 plots the number of vehicles, total value of

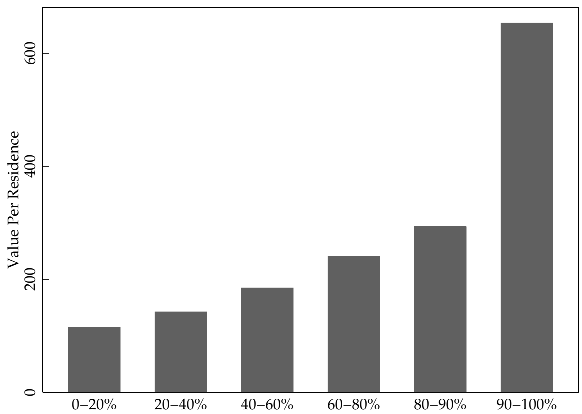
Figure 6: Housing Quantity and Quality by Percentile of Income



(a) Number of Residences



(b) Total Value of Residences (thousands of dollars)



(c) Value Per Residence (thousands of dollars)

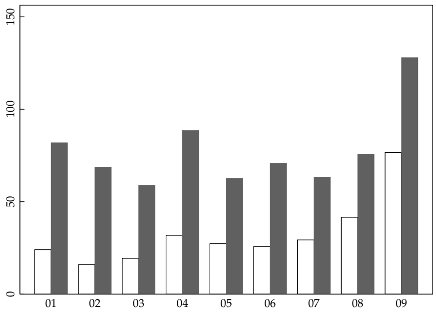
Notes: Source: SCF 2004.

vehicle purchases and unit value per vehicle across income classes. It shows that neither income elasticity of quantity nor income elasticity of quality is zero. As we move from lower income groups to higher income groups, both quantity and quality increase significantly.

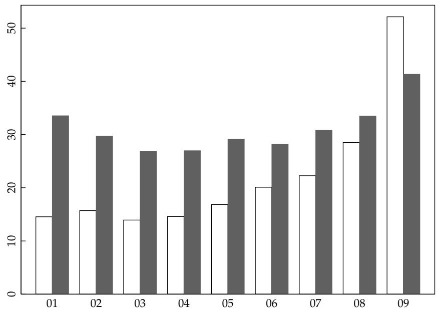
In addition to the four consumption goods noted above, Bils and Klenow (2001) also document the relative importance of income elasticity of quality and quantity for 66 durable goods in the CEX. Although they assume that the hedonic price function is linear, their results are consistent with some of the above evidence. For example, the income elasticity of quality of clothes is very low and the income elasticities of quantity and quality of vehicles are of the same magnitude.

In summary, the evidence presented shows that the income elasticities of quantity of food and housing are close to zero, the income elasticity of quality of clothes is close to zero and both are nonzero for vehicles. Some may argue that the observed two elasticities are equilibrium outcomes, which are endogenous. Hence, the observed patterns cannot be taken as primitives in the model. However, the observed elasticities, especially the zero income elasticity of quantity of food and housing, are not due to equilibrium outcomes, but instead, are due to the nature of the goods. For example, the daily calorie intake has to be within a certain range regardless of income and for convenience, a household usually has one primary residence. Finally, given the fact that the tradability of food and housing is generally lower than that of clothes and vehicles, it is reasonable to assume that x goods are tradable with the price normalized to 1 and z goods are non-tradable with a non-linear price function.

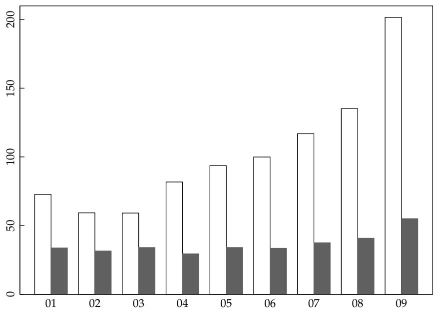
Figure 7: Total Expenditure and Unit Price for Different Types of Clothes in Dollars (CEX 2003).



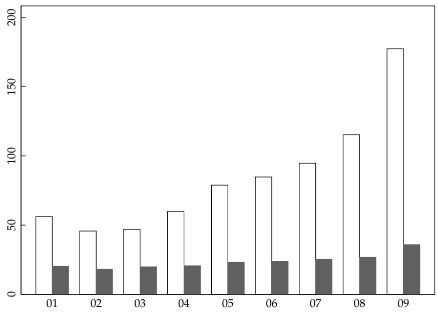
(a) Coats



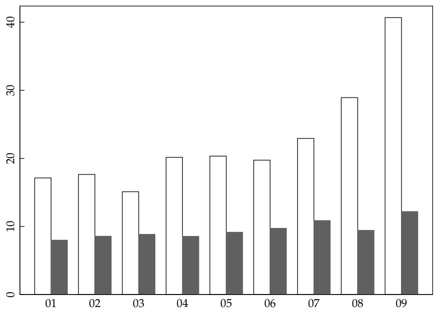
(b) Sweaters



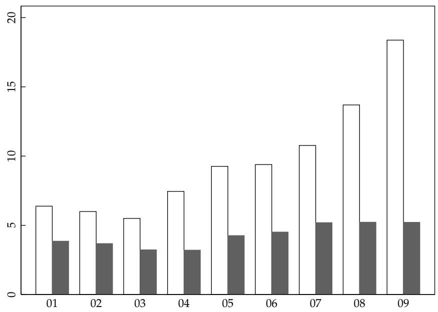
(c) Pants



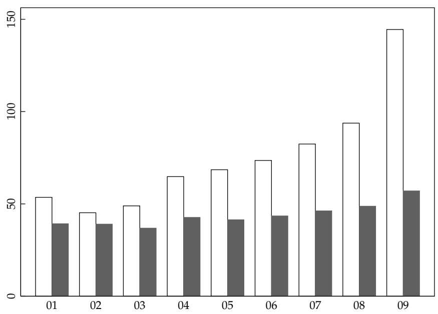
(d) Shirts



(e) Undergarments



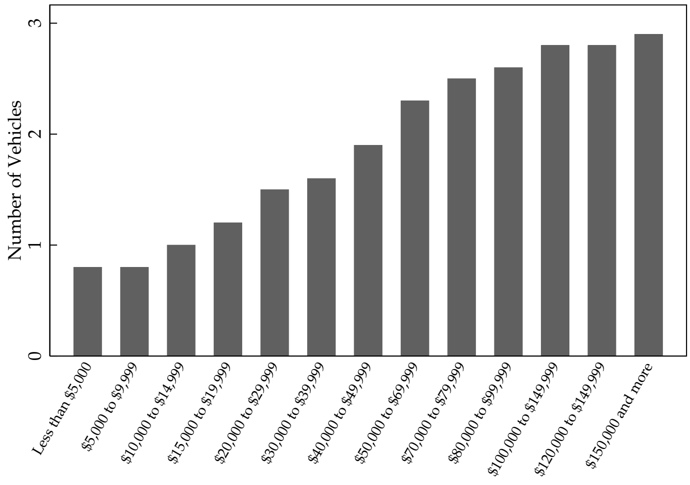
(f) Hosiery



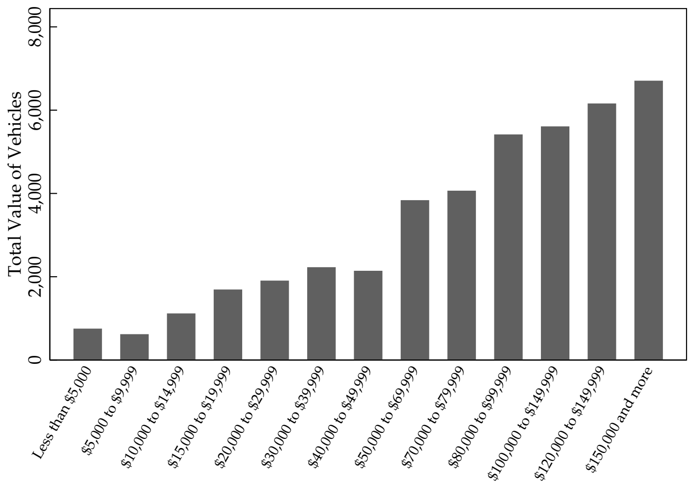
(g) Footwear

Notes: White bars denote total expenditure and dark bars denote unit price.

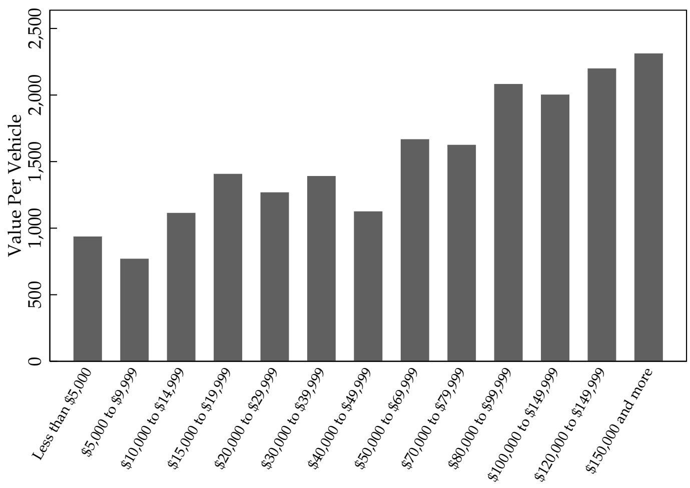
Figure 8: Vehicle Quantity and Quality by Percentile of Income



(a) Number of Vehicles



(b) Total Value of Vehicles (dollars)



(c) Value Per Vehicle (dollars)

Notes: Source: CEX 2007.

4.2. *The Impact of Income Distribution on the Price Levels of Individual Product Groups and Expenditure Shares*

Since the national price level is an average price level of disaggregate price levels weighted by expenditure shares. By examining how disaggregate price levels and expenditure shares vary with income distribution, we can trace out the main driving forces of the positive relationship between per capita income and the national price level, and, more importantly, the relationship between income inequality and the national price level.

Next, we investigate empirically how income distribution affects disaggregate price levels and expenditure shares. This leads to direct tests of the model's predictions in Proposition 1 and 3.

In the model, whether one good belongs to x or z will crucially determine how income distribution affects its price level and expenditure share. However, in practice, few goods are pure x or pure z. Nevertheless, we can quantify the degree to which a good belongs to x or z based on the features of these two types of goods. The prices of the x goods are assumed to be equalized across countries and the prices of the z goods are locally determined and related to local per capita income. We can therefore use the elasticity of a product's price with respect to per capita income, which is designated here as the quality index of the product, to measure whether the product is more like a x good or a z good.

As the most disaggregate level of the PPP data from the ICP is the basic heading level, we first compute the quality index for each basic heading by running cross-country regressions, by regressing the log of the price levels of one product on the log of countries' per capita income.

Then we regress the disaggregate price level on per capita income, the Gini coefficient and the product of per capita income and the Gini coefficient for each basic heading using the underlying PPP data from the ICP[8] and show how the estimation

[8]The dataset used here is from the ICP benchmark 2005, which provides disaggregate price indices and expenditure data at the basic heading level. They are the underlying data behind the national price

results vary from the basic headings with high quality index to the basic headings with low quality index.

$$\begin{aligned} log(Price_i) &= \beta_0 + \beta_1\, log(Per\ Capita\ Income) + \beta_2\, Gini \\ &\quad + \beta_3 log(Per\ Capita\ Income) \cdot Gini + \epsilon_i \end{aligned}$$

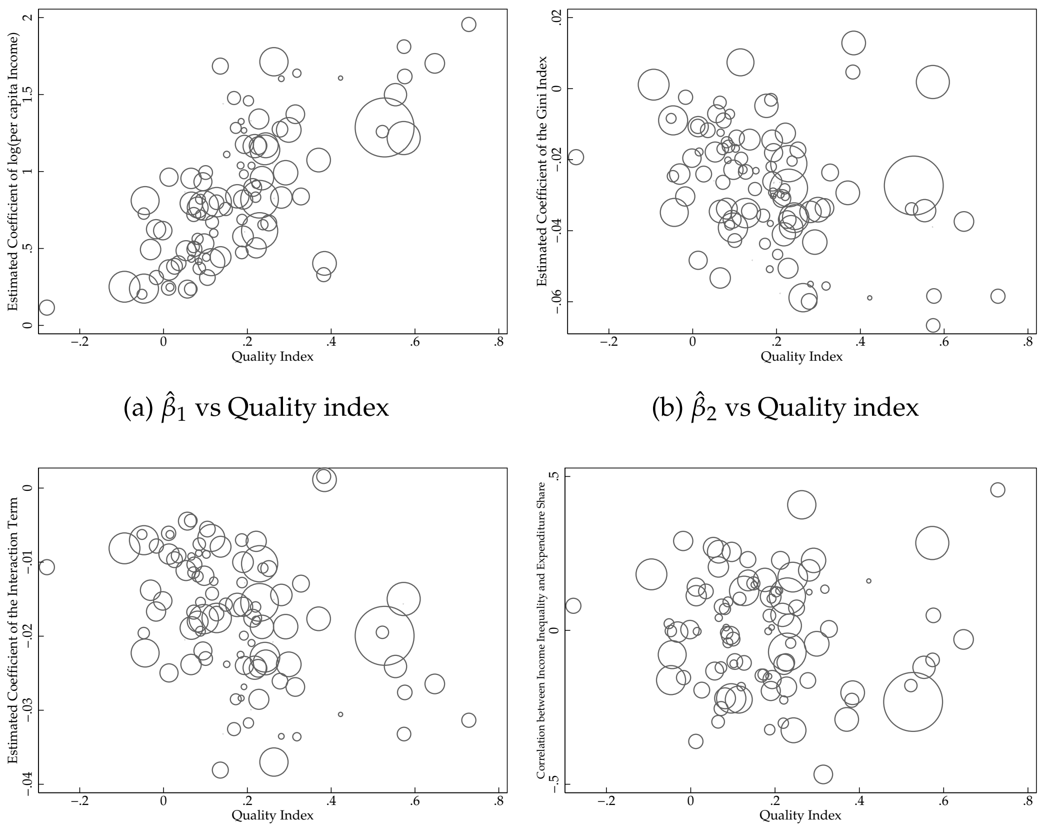
To do so, we plot the estimated coefficient from the above regression against the quality index to see how the latter affects the coefficient of per capita income, the Gini coefficient and the interaction term. Panel (a), (b) and (c) in Figure 9 plot these estimated coefficients, i.e. $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$, against the quality index.

Panel (a) of Figure 9 shows that there is a positive relationship between the quality index and the effect of per capita income on the national price level. Furthermore, as the quality index goes to zero, the effect of per capita income disappears, which implies that z goods are the main sources of the Penn effect, as the model has predicted. Moreover, we can see from Panel (b) and (c) that there is a negative relationship between the quality index and the estimated coefficients of the Gini index and the interaction term, i.e. as the quality index goes up, the impact of income inequality on the price level and the coefficient of the interaction term become more negative.

Moreover, as the quality index goes to zero, these two estimated coefficients also go to zero, i.e. income inequality has no impact on the price of x goods. Therefore the presence of z goods in the consumption bundle is the major driving force behind the negative coefficient of the Gini index and the interaction term in Regression (3), (4) and (5) of Table 1. These results are also consistent with the results in Proposition 3 and 4 that per capita income has a positive impact on the average price of z goods, income inequality has a negative impact on the average price of z goods and this impact is decreasing in per capita income. In summary, Panel (a), (b) and (c) show that income distribution has no impact on the price level of x goods (when the quality index is close to zero) and income distribution can affect the price level of z goods (when the quality

level in the Penn World Table. The basic headings which are classified as government consumption or investment are excluded from this study since the consumption of these categories is due to other reasons that are not supposed to be captured by this paper.

Figure 9: How the Quality Index Affects the Impact of Income Distribution on the Price Level of Individual Product Groups and Expenditure Share (Source: ICP 2005)



(a) $\hat{\beta}_1$ vs Quality index

(b) $\hat{\beta}_2$ vs Quality index

(c) $\hat{\beta}_3$ vs Quality index

(d) $Corr(Expenditure\ Share, Gini)$ vs Quality Index

Notes: The size of markers in the above scatter plots is proportional to the average expenditure share of each basic heading over all the countries in the ICP program.

index is high) as predicted in the theoretical model.

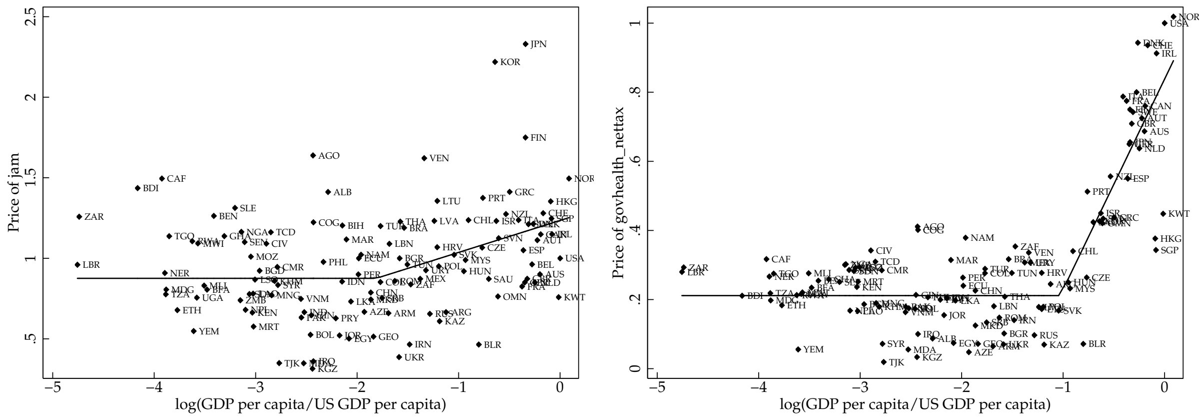
We can also test the model's prediction regarding how income inequality affects the expenditure share of z goods, which is the crucial mechanism generating the correlation between income inequality and the national price level. Panel (d) of Figure 9 plots the correlation coefficients between the expenditure share of each basic heading and income inequality against the quality index. It shows that there is a negative relationship between the quality index and the correlation between income inequality and expenditure share. More specifically, income inequality tends to have a positive impact on the expenditure share of x goods (when the quality index is low), while it tends to have a negative impact on the expenditure share of z goods (when the quality index is high). These empirical results are consistent with the model predictions in Proposition 1.

Thus, the empirical evidence at the disaggregate level is consistent with the model's mechanisms, through which income distribution affects the national price level.

5. Empirical Evidence III: Testing the Model at an Intermediate Level

We have used data at the basic heading level to show that the model's mechanism is consistent with empirical evidence. However, the data at the narrowly defined basic heading level is likely to be subject to idiosyncratic noise. This will make the relationship between income distribution and the price level less significant. Therefore, in order to more easily identify which component of the national price level is correlated with income distribution, it may be a good idea to test the relationship between income distribution and the price level at an intermediate level that is between the national level and the basic heading level. Our strategy to find the intermediate level is to first identify the archetypes from the B-S price-income relationship at the basic heading level and then aggregate the price levels of basic headings belonging to the same archetypes to a price index.

Our empirical strategy to identify the archetypes is, instead of assuming a stable relationship between the price levels of individual products and per capita income, to adopt a more agnostic approach by allowing for more flexibility in the parametric relationship between the price level and per capita income to accommodate the large variations in the relationship. This approach turns out to enable us to identify a clear and striking empirical pattern: for some products, the B-S relationship is weak and disperse. While for other products, the B-S relationship is highly nonlinear, which is best described as a spline relationship: within low- and middle- income countries, the relationship between per capita income and the price level is weakly positive, while within high-income countries, there is a sudden increase in the slope of the positive relationship.



(a) Jam (b) Government education net taxes on production

Figure 10: Two Archetypes of Price-Wealth Relationships

We illustrate the two types of relationship in Figure 10. Panel (a) shows the case of jam. On inspection, although there is a positive relationship, the explanatory power of per capita income for the price level is quite modest. In contrast, in panel (b) the price-income scatter plot for the governmental expenditure on education displays a striking nonlinear relationship with an almost zero slope within low- and mid- income country and a large slope among rich countries.

To facilitate the test of our later hypotheses, we need to develop a summary measure that captures the different quantitative forms of these relationships. It is not clear a priori how best to do this. We therefore explore a number of different approaches.

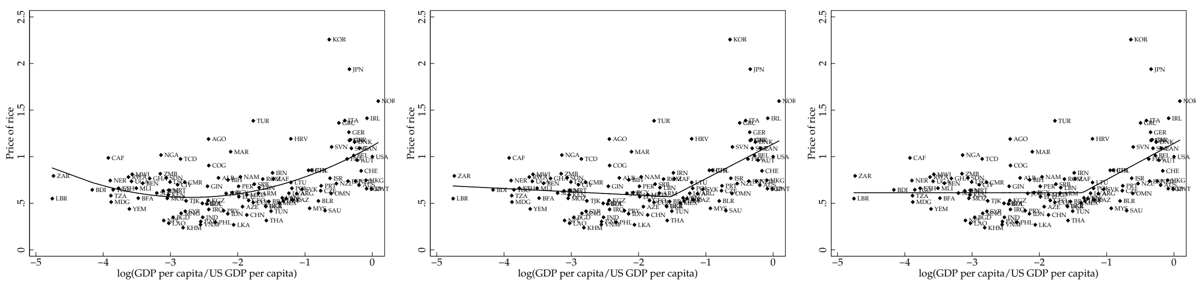
Approach 1 – a quadratic fit: we first use a quadratic function, i.e. a second-order polynomial to fit the data. The R^2 from the quadratic estimation is then chosen as the summary statistic of the degree to which each scatter plot displays a 'spline' relationship. The disadvantage of this approach is that it leads to occasional spurious results. For example, as shown in panel (a) of Figure 11, in the case of rice the quadratic fit generates a downward-sloping part on the left, which implies an insensible result: there is a negative relationship between the price level of rice and per capita income within poor countries.[9] Moreover, there is a large dispersion in the data points around the

[9] As Deaton (2010) pointed out, this is probably an outcome of comparing commonly available items in one country (e.g. Kellogg's cornflakes in the U.S.) with non-representative ones in a poor country, which are only available in some specialty stores of its big cities at very high prices. This will cause an

downward sloping part. This implies that this result is probably spurious.

Approach 2 – an unrestricted spline: we then try an unrestricted spline, i.e. a piecewise linear function with two segments. The two segments are defined by their intercept and slope parameters. Maximum likelihood estimation is used to determine the four parameters for each basic heading. The t-statistic of the slope coefficient of the second segment is used as the summary statistic for each scatter plot to indicate the degree of a 'spline' relationship. However, this second approach still cannot overcome the shortcomings of Approach 1. For example, in panel (b) of Figure 11 the unrestricted spline again yields a downward-sloping part that carries a large standard error, and is probably spurious.

Figure 11: Comparison of the Three Approaches for the Case of Rice



(a) Quadratic Fit (b) Unrestricted Spline Fit (c) Restricted Spline Fit

Approach 3 – a restricted spline: to avoid the arguably spurious results in the previous two approaches, we modify the second approach by restricting the slope of the first segment to be zero. Now the spline is determined by three parameters: the vertical position of the segment on the left, the horizontal position of the intersection (break point) and the slope of the segment on the right. Maximum likelihood estimation is used to estimate the three parameters. The t-statistic of the slope coefficient is used to measure the degree of a 'spline' type relationship, as in the second approach.

Approach 4 – the Box-Cox regression models. The Box-Cox regression models are used to generalize the linear model. In our case, in order to capture the nonlinear

upward bias in the price levels of poor countries. In particular, in order to avoid quality mismatching, the 2005 round of the ICP tried to match precisely specified goods and services, which is cited as an important reason to explain the differentially large increase in the price levels of poor countries relative to previous rounds.

price-wealth relationship, the Box-Cox model for the price level and GDP per capita is

$$Price_i^{(\lambda)} = \beta_0 + \beta_1 \ln(GDP\ per\ capita_i) + \epsilon_i$$

where the superscript (λ) is used to denote the Box-Cox transformation for a given value of λ, e.g.

$$y^{(\lambda)} = \frac{y^\lambda - 1}{\lambda} = \begin{cases} y - 1 & \text{if } \lambda = 1 \\ \ln y & \text{if } \lambda = 0 \end{cases}$$

As $price_i^{(1)} = price_i - 1$ and $price_i^{(0)} = \ln price_i$, the model also nests the following logarithm and semi-logarithm models as special cases:

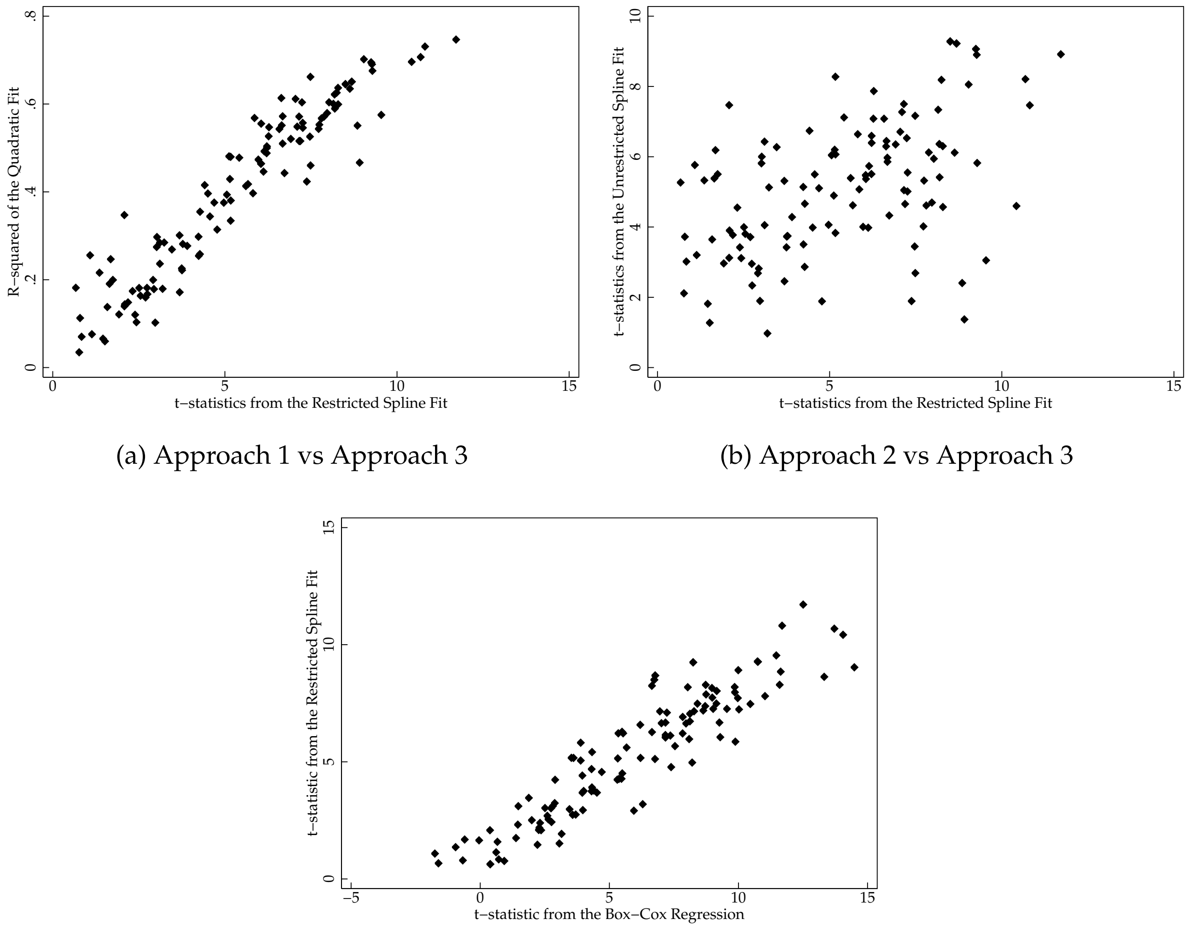
$$\ln(Price_i) = \beta_0 + \beta_1 \ln(GDP\ per\ capita_i) + \epsilon_i$$

$$Price_i = \beta_0 + \beta_1 \ln(GDP\ per\ capita_i) + \epsilon_i$$

The t-statistic of the estimated β_1 is used as the summary statistic of spline relationship in this approach.

We could also use other specifications to identify the spline relationship. However, the four approaches all work in the sense that the summary statistics do a good job in identifying the 'spline' relationship, which can be summarized by one robust measure. In the panel (a) of Figure 12 we plot the summary statistics from Approach 1 against those from Approach 3. In panel (b), the summary statistics from Approach 2 are plotted against those from Approach 3, and in panel (c) the summary statistics from Approach 3 are plotted against those from Approach 4. All plots show a positive relationship, which implies that the results from all the four approaches are consistent. However, as Approach 3 avoids the probably spurious result of a falling segment and its R^2 is consistently the highest among the four approaches, we use the summary statistic from Approach 3 as our main measure of the spline relationship.

Figure 12: Comparison of Summary Statistics of Three Approaches



(a) Approach 1 vs Approach 3

(b) Approach 2 vs Approach 3

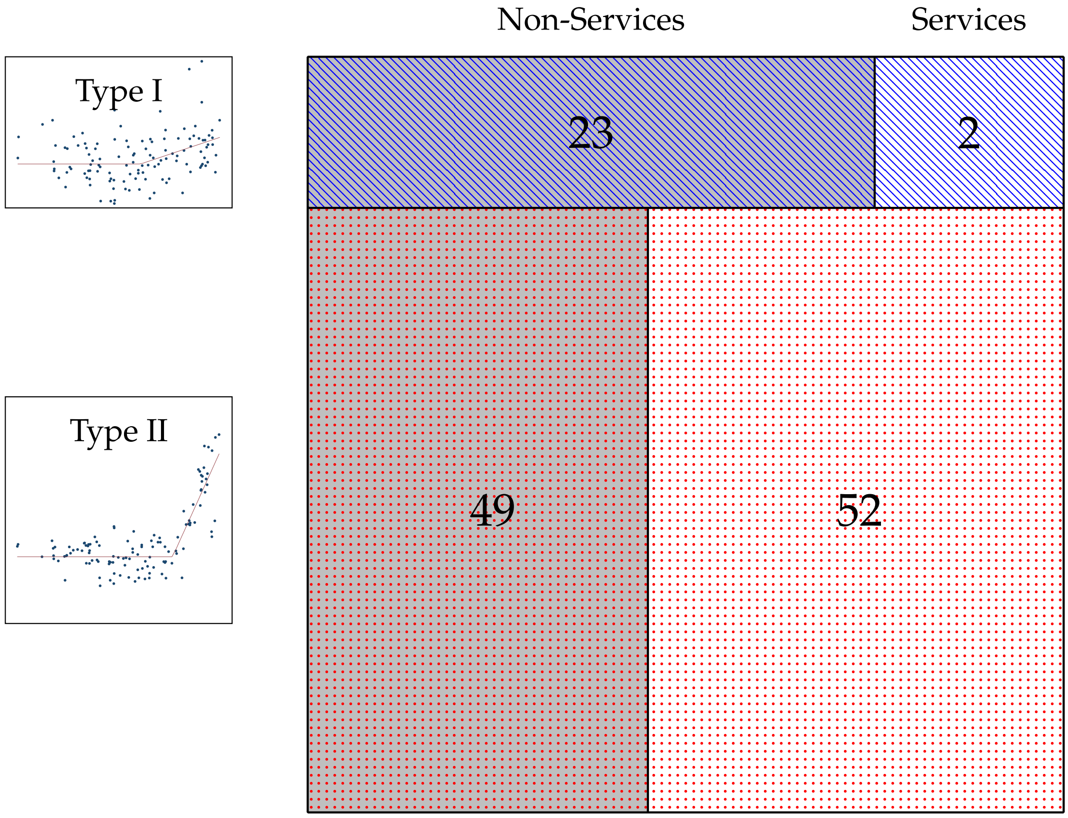
(c) Approach 3 vs Approach 4

An examination of the measure of the ′spline′ type relationship for each basic heading suggests a pattern: pure services tend to display the ′spline′ relationship, while the measure tends to be insignificant for manufactured products. This suggests looking for a further breakdown of all basic headings by our spline measure and by nature of its output (services or non-services) to examine the importance of this correlation.

To do so, we first discretize the B-S price wealth relationship by choosing a threshold for the measure of the spline relationship, i.e. the t-statistic of the slope coefficient: all the basic headings with a t-statistic less than 2.5 belong to type I, such as jam, while all those with a t-statistic greater than or equal to 2.5 belong to type II. Then, depending on whether the output is services or non-services, and on whether its price-wealth relationship is Type I or Type II, we allocate all the basic headings into a 2×2 table.

This is shown in Figure 13.

Figure 13: The Distribution of Basic Headings by Nature of Output (Services or Non-Services) and Price-Wealth Relationship



Notes: The basic headings with a price-income relationship of type I are represented by rectangles filled with north east lines. Those with a type II relationship are represented by those filled with dots. The nature of output of basic headings is indicated by the background color: services are marked by a white colour while non-services are marked by a gray one.

On inspection, we find few services basic headings displaying a Type I pattern except for the two in the top right corner. The top left rectangle contains 23 non-services basic headings, the price levels of which do not vary much with GDP per capita. This can be seen as empirical support for the Purchasing Power Parity (PPP) proposition: as most of non-services products are tradable, international arbitrage of the tradable non-services products can eliminate any price differentials across countries. However, the PPP proposition only holds for a small fraction of all basic headings. There are overall 101 basic headings in the bottom two rectangles with their price wealth relationship displaying the spline shape, which cannot be explained by the traditional theories of the national price level. In particular, 49 of them are non-services. This is in sharp contrast to the PPP proposition.

This suggests that it might be appropriate to think in terms of modelling the 'service' group (designated the 'S-group') separately from the manufactures group (designated the 'M-group').

Therefore, in order to identify the common statistical property of the B-S Price Wealth relationship of the S-group and M-group products and cancel out the idiosyncrasy of each basic heading, instead of working at the basic heading level, we will work at a more aggregate level - the group level. We will construct two aggregate price indexes for the two groups and study the B-S Price Wealth relationship at the group level. By this method, not only can we easily extract the common statistical properties of the two groups, we can also contrast their statistical properties and infer the distinctive features of each group. As it will become clear below, the above strategy greatly facilitates us in finding the empirical facts.

5.1. Re-estimating the Relationship

To formally show how income distribution affects the price level of M-group products and S-group products, we replicate the estimations in Table 1, but now we do so for the price level of S-group products, M-group products and the national price level for the year of 2005. The year 2005 is chosen as the sample year because the disaggregate price levels used to construct the price indices of the S-group products and the M-group products are from the ICP Benchmark Dataset 2005.[10]

All the estimation results are shown in Table 4. In the first three regressions, the relationships between the national price level and income distribution are consistent with what we have found in Section 3. The slope coefficient in Regression (1), i.e. the elasticity of the national price level with respect to per capita income is significantly

[10]Based on nature of its output, OECD has classified all the basic heading into six categories: ND (nondurable), SD (semi-durable), D (durable), S (service), IS (individual service), CS (collective service) and IG (investment goods). Based on this classification, we define the basic headings of services, individual services, and collective services as the S-group products, and define the rest basic headings as the M-group products.

positive. Including the Gini index as an additional regressor in Regression (2) only changes the results slightly. However, in Regression (3), when we include an interaction term, defined as the product of the Gini index and per capita income, both the Gini index and the interaction term become significantly negative. As argued in Section 3, this is due to the fact that the national price level depends both on per capita income and income inequality, and on their product. The lack of significance of the Gini index in Regression (2) is due to misspecification errors. The preferred specification of Regression (3) raises R^2 from 0.465 in Regression (1) to 0.554.

In Regression (4)-(9), we run the same regressions as above but using the disaggregate price levels of S-group products and M-group products. This can help us identify the sources of the results in Regression (1)-(3). On inspection, we can find that all the above qualitative results in Regression (1)-(3) in terms of the significance of and the sign of estimated coefficients and the improvement in R^2 also hold for the case of S-group products and M-group products. In Regression (4) and (7), per capita income has significantly positive impact on both the two price levels. Including the Gini index and the interaction term as additional regressors makes all estimated coefficients in Regression (6) and Regression (9) significant. The additional explanatory powers of the latter two regressors also increase R^2.

However, the magnitude of the results varies substantially between S-group products and M-group products. Firstly, the elasticity of the price level of S-group products with respect to per capita income, i.e. the coefficients of $\log(Y_j/Y_{U.S.})$ in Regression (4)-(6), are much higher than those of M-group products in Regression (7)- (9). For example, in Regression (4) the estimated elasticity is 0.489, which is six times the estimate in Regression (7). This is consistent with the fact that labour is the most important input in producing services as well as the fact that the service wage is nearly proportional to the average wage or GDP per capita. For M-group products, labour input plays less important role, so its price level will be less sensitive to the average wage.

Secondly, the R^2 in Regression (4) is 0.594. In Regression (6), adding the Gini index and the interaction term only increases the R^2 by 0.007. This again shows the crucial

Table 4: Income Distribution and the National Price Level: S-group products and M-group products

	Aggregate: $\log(P_j/P_{U.S.})$			S-group: $\log(P_{j,S}/P_{U.S.,S})$			M-group: $\log(P_{j,NS}/P_{U.S.,NS})$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\log(Y_j/Y_{U.S.})$	0.354***	0.361***	1.210***	0.489***	0.503***	1.464***	0.0811***	0.0886***	0.621***
	(0.0350)	(0.0390)	(0.188)	(0.0372)	(0.0416)	(0.199)	(0.0203)	(0.0207)	(0.0962)
Gini Index		0.0219	-3.96***		0.665	-3.84***		-0.298	-2.80***
		(0.544)	(1.00)		(0.581)	(1.06)		(0.289)	(0.511)
Gini Index $\times \log(Y_j/Y_{U.S.})$			-2.21***			-2.50***			-1.39***
			(0.482)			(0.509)			(0.246)
Constant	-0.152*	-0.149	1.293***	-0.593***	-0.834***	0.798**	-0.00628	0.144	1.049***
	(0.0787)	(0.205)	(0.366)	(0.0839)	(0.219)	(0.387)	(0.0458)	(0.109)	(0.187)
Observations	120	114	114	120	114	114	120	114	114
R^2	0.465	0.468	0.554	0.594	0.586	0.661	0.119	0.187	0.370

Standard errors in parentheses

$^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

role of GDP per capita in explaining the price level of S-group products. The Gini index can only provide a slight increase in explanatory power. However, a comparison of R^2s between Regression (7) and (9) shows a quite different picture. In Regression (7), given the small slope coefficient, the R^2 is only 0.119, but including per capita income and the Gini index increases R^2 to 0.370 in Regression (9). This implies that, compared with per capita income, income inequality matters much more for the price level of M-group products than that of S-group products. In other words, the non-service component of the national price level is more sensitive to income inequality than the service component.

5.2. *Interpreting the Results at the Group level*

Now our problems hinge on explaining two discrepancies between the model in Section 2 and the empirical evidence at the group level. On the one hand, income inequality can provide only a small amount of additional explanatory power for the price level of the S-group products on top of GDP per capita, while the model in Section 2 predicts that the impact of income inequality on the national price level should be mainly through its impact on the price level of nontradable vertically differentiated goods. On the other hand, income inequality is an important explanatory variable for the price level of the M-group products despite that the M-group products comprise mostly tradable goods, whose prices tend to be equalized across countries according the PPP proposition. We will reconcile these discrepancies in turn.

5.3. *Income Distribution and the Price Level of the S-group Products*

Firstly, the reconciliation of the discrepancy between the model and the empirical evidence in terms of the ability of income inequality in explaining the price level of the S-group products is related to the way the price level of services is measured in practice. Services, according to whether the market price is available, can be divided into market services and non-market services. The non-market services are predominantly provided by governments, which include collective government consumption (such as police, defence, fire-fighting and general government administration), health and education. Due to the fact that the bulk of these output is not sold on markets. So market prices are not available. And without prices, these outputs cannot be valued or compared satisfactorily. In the absence of measures of prices and output for these sectors, statisticians working on international comparisons - as well as national income accountants - have resorted to the use of inputs into the production of non-market services as proxies for output. Input costs are often available. The inputs include the sum of the wage costs of the employee involved in producing the services; the intermediate consumption of goods and services (materials used and rents, for example) and the services rendered by capital during the production process. As labour is the major input in producing non-market services, wage plays an essential role in determining the total cost or the price level reported by the International Comparison Program. Moreover, given the service wage usually changes proportionally with the average wage and the large share of non-market services in total services, GDP per capita should be a fairly good predictor for the input cost (or the price level) of services. The model in Section 2, however, examines the relationship between income distribution and the price level, but not the input costs, of nontradable vertically differentiated goods and predicts that income inequality should affect the price level of services. As the price level of services is largely unobservable, due to non-market services, the empirical result regarding the impact of income inequality on the price level of services is not a direct test of the model in Section 2. It only tests how income inequality affects the input cost instead of the price level.

5.4. Income Distribution and the Price Level of the M-group Products

Secondly, to explain why income inequality has significant impacts on the price level of the M-group products, we will show below that the model's mechanism for nontradable vertically differentiated goods to be affected by income inequality in the model can also be applied to tradable vertically differentiated goods. Although these goods are tradable in the traditional sense, i.e. transportation cost is low compared with unit values, we cannot simply apply the PPP proposition and claim that their price levels must be equalized across countries. This is because once tradable goods are vertically differentiated, the way tradable goods are priced and the way their price levels are compiled will change.

Once tradable goods become vertically differentiated or they can be produced at different levels of quality, a natural outcome will be each country's comparative advantage in producing a product with a certain level of quality: rich countries may be more dominant in the market of high quality products due to their advanced technology, while lower income countries may have a large market share in the low-end markets because of their cost advantage. The theoretical foundation and empirical supports was introduced in Sutton and Trefler (2011). The above specialization implies that these tradable vertically differentiated goods will be supplied by a group of producers, each of which is specialized to produce a product with a particular quality. Thanks to their comparative advantages, these producers can price discriminate against each country's market. Hence, to demonstrate how these producers price the tradable vertically differentiated goods according to each country's demand condition, we need to introduce quality to a model of 'Pricing to Market', which was introduced by Krugman (1987) to describe the practice of price discrimination across countries when international arbitrage is difficult or impossible. Our method is, therefore, to apply hedonic price model to an international context. This is very similar to the model in Section 2, but now we only need to replace nontradable vertically differentiated goods in the model by tradable vertically differentiated goods. As has been explained above, once tradable goods are differentiated by quality, their price levels are not determined internationally but will be linked with the local income distribution.

After the price function is determined, how their prices are compiled into a price index is also an important issue. Quality as a complicating factor for the B-S Price Wealth relationship has been studied in the literature. As quality cannot be perfectly controlled in compiling price indexes, the price index is just a simple average of individual prices without eliminating the impact of quality. Therefore, a higher quality will show up as a higher price level. For example, Schott (2004) showed that even within a Harmonized System (HS) - 10 category, quality is still an important explanatory variable for the U.S. important price. Empirical supports for the role of quality in the B-S Price Wealth relationship appeared in Goldberg and Verboven (2001), Hummels and Skiba (2004), Hallak (2006), Choi et al. (2009) and Imbs et al. (2010).

Given the above implications of vertical product differentiation, the next question is what mechanisms in the model cause the price level of the M-group products to be affected by income inequality. We can illustrate this by applying the model in Section 2 to the M-group products. Instead of assuming the consumption bundle is made up of both nontradable and tradable goods, it is now assumed that the consumption bundle includes only the M-group products, which consists of two types of goods: homogeneous goods and vertically differentiated goods, both of which are tradable. It is assumed that the consumption decision of each individual is to choose the quantity of the homogeneous goods and the quality of the vertically differentiated goods.

As the quality is constant for the homogeneous goods, there is no quality bias in its price index. However, the quality control problem in constructing the price level of vertically differentiated good is a practical issue that has yet been solved satisfactorily. As a result, quality can hardly be controlled in the price index and higher quality products imply higher prices. In addition, as implied by the standard hedonic price model in the literature such as in Rosen (1974) and Berry et al. (1995), the distribution of quality depends on the distribution of consumers' attributes, such as income distribution. Therefore, it means that in addition to per capita income, income inequality will matter for the aggregate price level of vertically differentiated goods by affecting its quality distribution. Similar to the model in Section 2, the new model will predict that a higher income inequality will imply a more convex price function of the vertically differentiated goods. Given a high enough elasticity of substitution between the homogeneous goods and the differentiated goods, consumers will respond to the change in income inequality by lowering the expenditure share on differentiated goods. As its quality cannot be controlled, the lower expenditure will be translating to a lower price level.

With a constant price level of the homogeneous goods, the aggregate price level of the M-group products will be lower. Moreover, the aggregate price level of the M-group products is an weighted average of the price levels of homogeneous goods and vertically differentiated goods, so the product of the expenditure share and the price level of differentiated goods is a crucial component in the formula of the aggregate price level. The higher per capita income in rich countries, which implies a higher price level of differentiated goods, can interact with income inequality and magnify the negative impact of income inequality on the aggregate price level. This can be empirically supported by the significantly negative coefficient of the interaction term in Regression (9) of Table 4. In other words, the underprediction caused by failing to use income inequality is especially severe in rich countries. Hence, if we take into account the impact of income inequality, we can explain a much higher fraction of variations in the price level of manufactures.

6. Conclusion

This paper provides a new candidate explanation for the B-S relationship, which is based on an appeal to mismeasured quality. The key assumption is that income elasticity of quality is non-negligible and tends to be higher for nontraded goods. The model both implies the B-S relationship, and also implies that income inequality has a positive impact on the national price level within countries with lower per capita income, while within countries with higher per capita income, the impact is negative. We show that this second prediction is also consistent with empirical evidence.

Testing the model's predictions at a disaggregate level shows that income inequality explains a higher proportion of variations in the price level of manufactures than in the price level of services, suggesting that both tradable and nontradable goods are important in forming the relationship between income distribution and the price level.

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A. Appendix

A.1. Proof of Proposition 3

Since $\tau > 1$,

$$\frac{\partial \overline{p}}{\partial \mu} = \frac{\beta}{\alpha \frac{k_z + \gamma \frac{1}{\tau - 1}}{\frac{\frac{1}{Gini}+1}{2} + \frac{1}{\tau - 1}} + \beta} > 0$$

$$\frac{\partial \overline{p}}{\partial Gini} = \mu \frac{-\alpha\beta}{(\alpha \frac{k_z + \gamma \frac{1}{\tau - 1}}{\frac{\frac{1}{Gini}+1}{2} + \frac{1}{\tau - 1}} + \beta)^2} \frac{k_z + \gamma \frac{1}{\tau - 1}}{(\frac{\frac{1}{Gini}+1}{2} + \frac{1}{\tau - 1})^2} \frac{1}{2(Gini)^2} < 0$$

Therefore,

$$e_{\overline{p},\mu} = \frac{\partial \overline{p}}{\partial \mu} \frac{\mu}{\overline{p}} > 0$$

$$e_{\overline{p},Gini} = \frac{\frac{\partial \overline{p}}{\overline{p}}}{\partial Gini} = \frac{\partial \overline{p}}{\partial Gini} \frac{1}{\overline{p}} < 0$$

That is the elasticity of the average price level of the z goods with respect to per capita income is positive and its semi-elasticity with respect to income inequality is negative. Q.E.D.

A.2. *Proof of Proposition 4*

$$\frac{\partial \overline{p}}{\partial \mu \partial Gini} = \frac{-\alpha\beta}{(\alpha \frac{k_z+\gamma\frac{1}{\tau-1}}{\frac{\frac{1}{Gini}+1}{2}+\frac{1}{\tau-1}} + \beta)^2} \frac{k_z+\gamma\frac{1}{\tau-1}}{(\frac{\frac{1}{Gini}+1}{2}+\frac{1}{\tau-1})^2} \frac{1}{2(Gini)^2} < 0$$

According to Young's theorem, $\frac{\partial \overline{p}}{\partial \mu \partial Gini} = \frac{\partial \overline{p}}{\partial Gini \partial \mu}$. Therefore, the absolute value of $\frac{\partial \overline{p}}{\partial Gini}$ is increasing in μ. And $\frac{\partial \overline{p}}{\partial \mu}$ is decreasing in $Gini$. Q.E.D.

A.3. *Proof of Proposition 5*

Given the definition of the aggregate price level as the Paasche index,

$$P_P = 1\frac{\alpha d}{\alpha d+\beta} + \frac{\overline{p}}{\overline{p}_0}\frac{\beta}{\alpha d+\beta}.$$

$$= (1-\frac{\beta}{\alpha d+\beta}) + \frac{\mu\frac{\beta}{\alpha d+\beta}}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\frac{\beta}{\alpha d+\beta}. \tag{A.1}$$

we have

$$\frac{\partial P_P}{\partial \mu} = (\frac{\beta}{\alpha d+\beta})^2 \frac{1}{\mu_0\frac{\beta}{\alpha d_0+\beta}} > 0 \tag{A.2}$$

$$\frac{\partial P_P}{\partial Gini} = \frac{\beta}{(\alpha d+\beta)^2}\alpha\frac{\partial d}{\partial Gini} + \frac{\mu}{\mu_0\frac{\beta}{ad_0+\beta}}\beta^2(-2)(\alpha d+\beta)^{-3}\alpha\frac{\partial d}{\partial Gini}$$

$$= [\frac{\alpha\beta}{(\alpha d+\beta)^2} - 2\frac{\mu}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\alpha\beta^2(\alpha d+\beta)^{-3}]\frac{\partial d}{\partial Gini}$$

$$= [1-2\frac{\mu\frac{\beta}{\alpha d+\beta}}{\mu_0\frac{\beta}{\alpha d_0+\beta}}]\frac{\alpha\beta}{(\alpha d+\beta)^2}\frac{\partial d}{\partial Gini} \tag{A.3}$$

Hence, the impact of per capita income on the Paasche index is positive while the impact of income inequality on the Paasche index depends on the country's income distribution relative to the base country, i.e. the U.S.. $\frac{\partial P_P}{\partial Gini}$ for different combinations of per capita income and income inequality using the value of parameters from Table A.5 is plotted in Figure A.14. It can be seen that the sign of $\frac{\partial P_P}{\partial Gini}$ crucially depends on the level of per capita income. With low per capita income, income inequality has a positive impact on the national price level. On the other hand, with high per capita income, income inequality will have a negative impact on the national price level.

Table A.5: Calibration

Baseline specification	
$\alpha = 0.1$	share of homogeneous goods
$\beta = 0.9$	share of differentiated goods
$\epsilon = 1$	elasticity of substitution
$A_z = 1$	productivity parameter
$\tau = 2$	quantity elasticity of production cost
$\gamma = 2$	quality elasticity of production cost
$k_c = 1.7255$	power parameter in Pareto distribution of income
$c_m = 17862$	minimum income
$k_z = 2$	power parameter in Pareto distribution of quality
$z_m = \frac{\sqrt{2}}{2}$	minimum quality

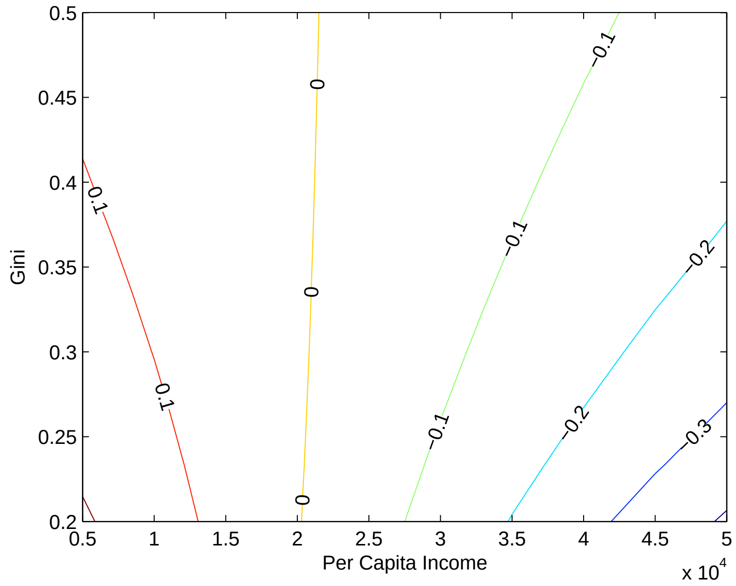
Moreover,

$$\frac{\partial P_P}{\partial\mu\partial Gini} = -2\frac{1}{\mu_0\frac{\beta}{\alpha d_0+\beta}}\alpha\beta^2(\alpha d+\beta)^{-3}\frac{\partial d}{\partial Gini} < 0. \tag{A.4}$$

Thus, the effect of per capita income on the aggregate price level ($\frac{\partial P_P}{\partial \mu}$) is decreasing in income inequality and the effect of income inequality ($\frac{\partial P_P}{\partial Gini}$) is decreasing in per capita income μ.

Since the elasticity of the aggregate price level with respect to per capita income $e_{P_P,\mu} \equiv \frac{\partial P_P}{\partial \mu}\frac{\mu}{P_P}$ has the same sign as $\frac{\partial P_P}{\partial \mu}$ and the semi-elasticity of the aggregate price level with respect to income inequality $e_{P_P,Gini} \equiv \frac{\partial P_P}{\partial Gini}\frac{1}{P_P}$ has the same sign as $\frac{\partial P_P}{\partial Gini}$, it is easy to show that $e_{P_P,\mu}$ is positive and the sign of $e_{P_P,Gini}$ depends on per capita income of the country in question relative to the U.S.. With a low enough per capita income $e_{P_P,Gini}$ is positive, while $e_{P_P,Gini}$ is negative with a high level of per capita income.

Figure A.14: The Contour of the Effect of Income Inequality on the Paasche Index ($\frac{\partial P_P}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality



Notes: The base country income distribution is calibrated using U.S. data in 2005.

Furthermore,

$$\frac{\partial e_{P_P,\mu}}{\partial Gini} = \frac{\partial[\frac{\partial P_P}{\partial \mu}\frac{\mu}{P_P}]}{\partial Gini}$$

$$= \frac{\partial P_P}{\partial \mu \partial Gini}\frac{\mu}{P_P} + \frac{\partial P_P}{\partial \mu}\frac{\mu}{-P_P^2}\frac{\partial P_P}{\partial Gini} \tag{A.5}$$

Substituting Equation (A.1), (A.2), (A.3) and (A.4) into Equation (A.5), we can obtain

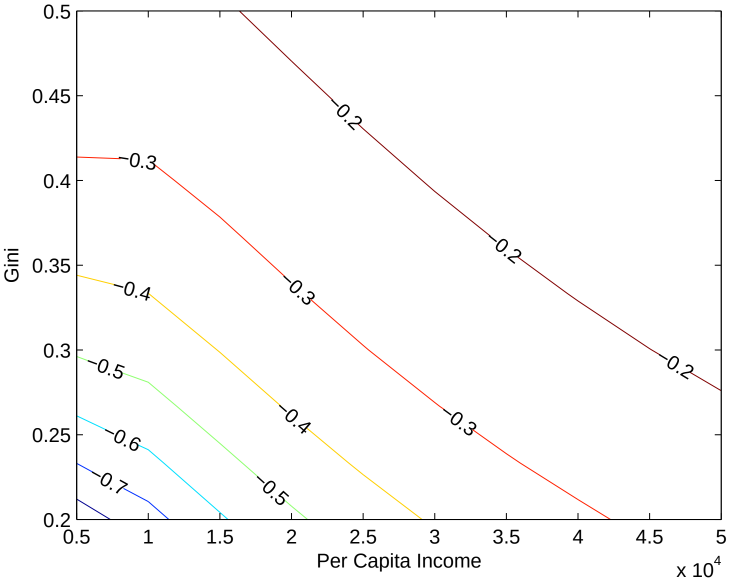
$$\frac{\partial e_{P_P,\mu}}{\partial Gini} = \frac{\mu}{P_P}\frac{\partial d}{\partial Gini}(\alpha d+\beta)^{-3}\frac{1}{\mu_0 \frac{\beta}{\alpha d_0+\beta}}\alpha\beta^2\{-2-\frac{\beta}{\alpha d+\beta}[1-2\frac{\overline{P}}{\overline{P}_0}]\frac{1}{P_P}\}$$

Therefore, the condition for $\frac{\partial e_{P_P,\mu}}{\partial Gini} < 0$ is

$$\frac{\beta}{\alpha d+\beta}(2\frac{\overline{P}}{\overline{P}_0}-1)\frac{1}{P_P} < 2$$

Intuitively, we can notice that this condition can be satisfied as long as the income distribution is not too far away from that of the U.S.. For example, when the income distribution of the country in question is similar to that of the base country, the U.S. , in which case both $\frac{\overline{P}}{\overline{P}_0}$ and P_P are around 1, the left hand side will be around $\frac{\beta}{\alpha d+\beta}$, which is much less than 2.

Figure A.15: The Contour of the Effect of Income Inequality on $e_{P_P,\mu}$ ($\frac{\partial e_{P_P,\mu}}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality



Notes: The base country income distribution is calibrated using U.S. data in 2005.

To check the sign of $\frac{\partial e_{P_P,\mu}}{\partial Gini}$ more generally, its value for different combinations of per capita income and income inequality is plotted in Figure A.15. The Figure shows that $\frac{\partial e_{P_P,\mu}}{\partial Gini}$ is negative for all possible combinations of per capita income and income inequality. Hence, the elasticity of the aggregate price level with respect to per capita income is decreasing in income inequality. Similarly,

$$\frac{\partial e_{P_P,Gini}}{\partial \mu} = \frac{\partial[\frac{\partial P_P}{\partial Gini}\frac{1}{P_P}]}{\partial \mu}$$

$$= \frac{\partial P_P}{\partial Gini \partial \mu}\frac{1}{P_P} + \frac{\partial P_P}{\partial Gini}\frac{-1}{P_P^2}\frac{\partial P_P}{\mu}$$

$$= \frac{1}{P_P}[\frac{\partial P_P}{\partial Gini \partial \mu} - \frac{\partial P_P}{\partial Gini}\frac{1}{P_P}\frac{\partial P_P}{\mu}] = \mu \frac{\partial e_{P_P,\mu}}{\partial Gini}$$

Since $\frac{\partial e_{P_P,Gini}}{\partial \mu}$ has the same sign as $\frac{\partial e_{P_P,\mu}}{\partial Gini}$, the semi-elasticity of the aggregate price level with respect to income inequality is decreasing in per capita income.

If we define the aggregate price level as the Laspeyres index, then

$$P_L = (1 - \frac{\beta}{\alpha d_0 + \beta}) + \frac{\mu^{\frac{\beta}{\alpha d + \beta}}}{\mu_0^{\frac{\beta}{\alpha d_0 + \beta}}} \frac{\beta}{\alpha d_0 + \beta}$$

$$= (1 - \frac{\beta}{\alpha d_0 + \beta}) + \frac{\mu}{\mu_0}\frac{\beta}{\alpha d + \beta} \tag{A.6}$$

Therefore,

$$\frac{\partial P_L}{\partial \mu} = \frac{\beta}{\mu_0(\alpha d + \beta)} > 0 \tag{A.7}$$

$$\frac{\partial P_L}{\partial Gini} = \frac{\mu}{\mu_0}\frac{-\beta}{(\alpha d + \beta)^2}\alpha \frac{\partial d}{\partial Gini} < 0 \tag{A.8}$$

$$\frac{\partial P_L}{\partial \mu \partial Gini} = \frac{1}{\mu_0}\frac{-\beta}{(\alpha d + \beta)^2}\alpha \frac{\partial d}{\partial Gini} < 0 \tag{A.9}$$

Hence, the impact of per capita income on the Laspeyres index is positive while the impact of income inequality on the Laspeyres index is negative. Moreover, the impact of per capita income on the Laspeyres index is decreasing in income inequality and the impact of income inequality on the Laspeyres index is decreasing in per capita income.

Since the elasticity of the aggregate price level with respect to per capita income $e_{P_L,\mu} \equiv \frac{\partial P_L}{\partial \mu}\frac{\mu}{P_L}$ has the same sign as $\frac{\partial P_L}{\partial \mu}$ and the semi-elasticity of the aggregate price level with respect to income inequality $e_{P_L,Gini} \equiv \frac{\partial P_L}{\partial Gini}\frac{1}{P_L}$ has the same sign as $\frac{\partial P_L}{\partial Gini}$, it is easy to show that $e_{P_L,\mu}$ is positive and the sign of $e_{P_L,Gini}$ is negative.

Furthermore,

$$\frac{\partial e_{P_L,\mu}}{\partial Gini} = \frac{\partial[\frac{\partial P_L}{\partial \mu}\frac{\mu}{P_L}]}{\partial Gini}$$

$$= \frac{\partial P_L}{\partial \mu \partial Gini}\frac{\mu}{P_L} + \frac{\partial P_L}{\partial \mu}\frac{\mu}{-P_L^2}\frac{\partial P_L}{\partial Gini}$$

$$= \frac{\mu}{P_L}[\frac{\partial P_L}{\partial \mu \partial Gini} - \frac{\partial P_L}{\partial \mu}\frac{1}{P_L}\frac{\partial P_L}{\partial Gini}] \tag{A.10}$$

Substituting Equation (A.6), (A.7), (A.8) and (A.9) into Equation (A.10), we can obtain

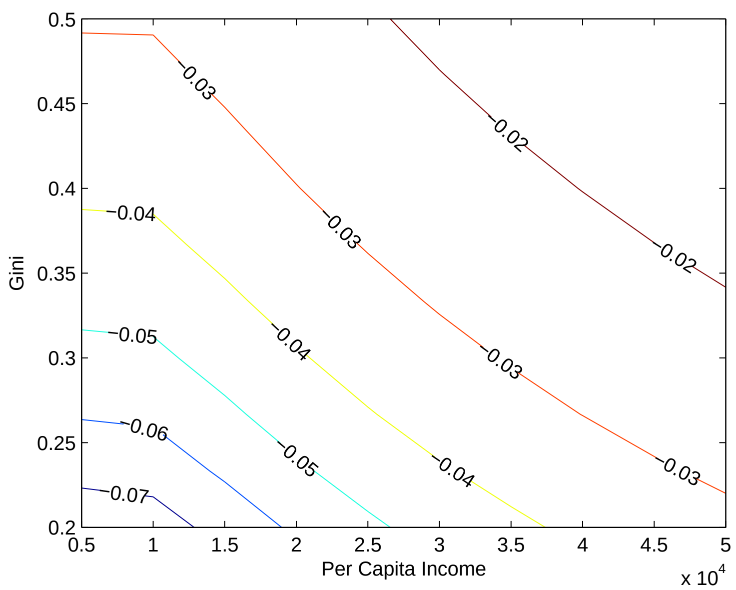
$$\frac{\partial e_{P_L,\mu}}{\partial Gini} = \frac{\mu}{P_L}\frac{1}{\mu_0}\frac{\alpha\beta}{(\alpha d+\beta)^2}\frac{\partial d}{\partial Gini}[-1+\frac{\beta}{\alpha d+\beta}\frac{\mu}{\mu_0}\frac{1}{P_L}]$$

Therefore, the condition for $\frac{\partial e_{P_L,\mu}}{\partial Gini} < 0$ is

$$\frac{\beta}{\alpha d+\beta}\frac{\mu}{\mu_0}\frac{1}{P_L} < 1$$

Intuitively, this condition can be satisfied as long as the income distribution of the country in question is not far away from that of the base country the U.S.. For example, when the income distribution is similar to that of the U.S. , in which case both $\frac{\mu}{\mu_0}$ and P_L are around 1, the left hand side will be around $\frac{\beta}{\alpha d+\beta}$, which is less than 1.

Figure A.16: The Contour of Effect of Income Inequality on $e_{P_L,\mu}$ ($\frac{\partial e_{P_L,\mu}}{\partial Gini}$) for Different Combinations of Per Capita Income and Income Inequality



Notes: The base country income distribution is calibrated using U.S. data in 2005.

To check the sign of $\frac{\partial e_{P_L,\mu}}{\partial Gini}$ more generally, its value for different combinations of per capita income and income inequality is plotted Figure A.16. The Figure shows that $\frac{\partial e_{P_L,\mu}}{\partial Gini}$ is negative for all possible combinations of per capita income and income inequality. Hence, the elasticity of the aggregate price level with respect to per capita income is decreasing in income inequality. Similarly,

$$\frac{\partial e_{P_L,Gini}}{\partial \mu} = \frac{\partial [\frac{\partial P_L}{\partial Gini} \frac{1}{P_L}]}{\partial \mu}$$

$$= \frac{\partial P_L}{\partial Gini \partial \mu} \frac{1}{P_L} + \frac{\partial P_L}{\partial Gini} \frac{-1}{P_L^2} \frac{\partial P_L}{\partial \mu}$$

$$= \frac{1}{P_L} [\frac{\partial P_L}{\partial Gini \partial \mu} - \frac{\partial P_L}{\partial Gini} \frac{1}{P_L} \frac{\partial P_L}{\partial \mu}]$$

$$= \frac{1}{\mu} \frac{\partial e_{P_L,\mu}}{\partial Gini}$$

Since $\frac{\partial e_{P_L,Gini}}{\partial \mu}$ has the same sign as $\frac{\partial e_{P_L,\mu}}{\partial Gini}$, the semi-elasticity of the aggregate price level with respect to income inequality is decreasing in per capita income. Q.E.D.