Variance Risk Premium and VIX Pricing: A Simple GARCH Approach

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University of Finance and Economics, by Huaxi Futures Co., Ltd., and by Tri-Spring Steel
Trades Co., Ltd.
Abstract: This study proposes a simple GARCH method for pricing VIX. Closed-form formulas for VIX are derived for GARCH(1,1), asymmetric GJR GARCH(1,1), and Heston-Nandi GARCH(1,1) models. With the empirical GARCH parameters estimated from 3500 daily returns for the S&P 500 Index, these formulas under-price VIX by 12.0~33.9% on average for the two periods of January 1996-September 2003 and September 2003-January 2012. This underestimation could be interpreted as variance risk premium. On the other hand, the parameters in these formulas can be calibrated by the market VIX of the previous trading day to obtain the risk-neutral measure. For the same two periods, the risk-neutral parameters price VIX accurately with a mean pricing error of not more than 0.2%.

Keywords: Variance Risk Premium; VIX Pricing; VIX formulas; GARCH(1,1); GJR GARCH; Heston-Nandi GARCH

JEL Classification: G13, G12


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1. INTRODUCTION

The CBOE Volatility Index (VIX), proposed by Whaley (1993) and introduced in 1993, has become the standard gauge of the investor fear and market sentiment. It is befittingly referred to as the “fear index” by popular news media, such as the New York Times and the Wall Street Journal. Nowadays, CBOE trades VIX futures, VIX options, VIX Binary Options, and Mini-VIX futures.

As a forecast for the 30-day volatility of the S&P 500 Index, VIX and its pricing (or its own forecasting) have not overlooked by academia. Using the Heston stochastic variance model, Zhang and Zhu (2006) initiated the study of VIX futures pricing, but did not try to forecast VIX. Closed-form formulas for the fair value of VIX futures were derived for several stochastic variance models with jumps in both the asset and variance processes (Lin, 2007). Duan and Yeh (2010) utilized the maximum likelihood method to estimate the parameters of stochastic volatility models with or without jumps. The Heston stochastic variance model was employed by Zhang and Huang (2010) in a study of the CBOE S&P500 three-month variance futures. VIX futures were investigated by assuming a square root mean-reverting process for the variance (Zhang, Shu & Brenner, 2010). Zhu and Lian (2011) obtained a closed-form exact solution for VIX futures in a stochastic volatility model with simultaneous jumps in both the asset and volatility processes. All these researches mentioned above deal with continuous-time variance or volatility models, the parameters of which are calibrated by the market VIX level of a previous trading day.
The calibrated (risk-neutral) parameters are then used to forecast the price of futures.

As a discrete time model for volatility, GARCH models seem to be a natural choice for studying VIX, but nobody appears to have paid serious attention to it so far. Barone-Adesi et al. (2008) alluded to the pricing of VIX briefly, when they proposed the filtered historical simulation GJR GARCH method for pricing European options. They used the market prices of S&P 500 Index options from a previous trading day to calibrate GARCH parameters in order to obtain the risk-neutral GARCH model. Their idea of path simulations was later applied to VIX pricing with some more details (Byun & Min, 2012). VIX formulas under the empirical measure for five GARCH models were presented by Hao and Zhang (2010), who did not try to obtain the risk-neutral parameters for forecasting VIX.

This paper proposes a simple GARCH based VIX pricing method. For GARCH(1,1), GJR, and Heston-Nandi variance models, simple closed-from formulas are derived first. Then instead of using options market prices for the calibration of parameters, the new approach utilizes directly the market VIX of the previous trading day to obtain risk-neutral GARCH parameters. With the risk-neutral parameters, the VIX formulas can be applied to VIX pricing. Later, empirical investigations are carried out for the periods of 2 January 1996 - 19 September 2003 and of 22 September 2003 - 31 January 2012. The empirical results reveal the variance risk premium under the empirical measure and provide adequate proof of concept for the new pricing method. Finally, the paper concludes with comments.
2. VIX FORMULAS UNDER GARCH(1,1) MODELS

Several GARCH(1,1) models can be written as follows under either historical or risk-neutral measures:

\[ \ln(S_t / S_{t-1}) = \mu + \varepsilon_t \]

\[ v_t = \omega + \beta v_{t-1} + u_{t-1} \]  \hspace{1cm} (1),

where \( v_t = \sigma_t^2 \) is the variance, and \( \varepsilon_t = \sigma_t z_t \) is the innovation for date \( t \). Here \( z_t \) is either a standard normal variable or the empirical random variable with a mean of zero and a variance of one. For GARCH(1,1) or G11 hereafter, \( u_t = \alpha \varepsilon_t^2 \) (Hull, 2009); for asymmetric GJR GARCH(1,1) or GJR hereafter, \( u_t = (\alpha + \gamma I[z_t < 0])\varepsilon_t^2 \), where \( I[z_t < 0] \) is the indicator function (Barone-Adesi, Engle & Mancini, 2008); for HN GARCH(1,1) or HN hereafter, \( u_t = \alpha(z_t - \gamma \sigma_t)^2 \) (Heston & Nandi, 2000).

By definition, \( z_t \) and \( \sigma_t \) are independent, and the conditional expectations are \( E_t[z_{t+1}] = 0 \), \( E_t[z_{t+1}^2] = 1 \), \( E_t[z_{t+1} | \sigma_{t+1}] = 0 \), and \( E_t[v_{t+1}] = v_{t+1} \). Further, \( E_t[z_{t+1}^2 | I[z_t < 0]] = 0.5 \). Therefore using GJR as an example, one has:

\[ E_t[v_{t+k}] = \omega + E_t[\beta v_{t+k-1} + (\alpha + \gamma I[z_{t+k-1} < 0])\varepsilon_{t+k-1}^2] \]

\[ = \omega + E_t[(\beta + E_t[\gamma I[z_{t+k-1} < 0])\varepsilon_{t+k-1}^2)]v_{t+k-1} \]

\[ = \omega + \xi E_t[v_{t+k-1}] \]

where \( \xi = \alpha + \beta + 0.5 \gamma \). Let \( \omega = (1 - \xi) V_L \), where \( V_L \) represents the long-term average variance. Then from the above expression it is easy to show (following Hull (2009)) that:

\[ E_t[v_{t+k} - V_L] = \xi E_t[v_{t+k-1} - V_L] \]

\[ = \xi^{k-1}[v_{t+1} - V_L] \]

Now the CBOE 30-day volatility index VIX can be computed according to
Equation (12)\(^1\) of Barone-Adesi et al. (2008) as:

\[\text{VIX}_t^2 = 100^2 \times 365 + 30 \times (\sum_{k=1}^{30} E[v_{t+k}]) \times 252 + 365\]

\[= 84000[30V_L + (v_{t+1} - V_L) \sum_{k=1}^{30} \xi^{k-1}]\]

\[= 84000[30V_L + \frac{1 - \xi^{30}}{1 - \xi}]

\[\xi = \alpha + \beta + 0.5\gamma, V_L = \omega/(1 - \xi) \quad \text{(for GJR)} \quad (3).

Note that Equation (2) has exactly the same linear form as those given by the continuous-time stochastic volatility models (Zhang & Zhu, 2006; Lin, 2007; Zhu & Lian, 2011). Similar but a little bit more complicated results for GARCH(1,1), EGARCH, TGARCH, AGARCH and CGARCH models were derived by Hao and Zhang (2010). One nice feature of Equation (2) is its independence of the parameters of the “stock” process, while both the results of continuous-time models and Hao and Zhang (2010) are not.

Interestingly Equation (2) is also correct for G11 and HN given the following parameters:

\[\xi = \alpha + \beta, V_L = \omega/(1 - \xi) \quad \text{(for G11)} \quad (4),

\[\xi = \alpha \gamma^2 + \beta, V_L = (\omega + \alpha)/(1 - \xi) \quad \text{(for HN)} \quad (5).

With only GARCH parameters \(\omega, \alpha, \beta\) and \(\gamma\) (for GJR and HN only) under either the empirical or risk-neutral measure, VIX can then be computed directly using Equation

\(^1\) GARCH variance is measured in trading days, while VIX is computed in calendar days. Therefore, the right hand-side of Equation (12) of Barone-Adesi et al. (2008) shall be multiplied by a factor of 252/365.
(2) and (3), (4), or (5).

3. VARIANCE RISK PREMIUM AND VIX-CALIBRATED GARCH PARAMETERS

Obviously given GARCH parameters estimated from the historical prices of the underlying asset, VIX can be computed via Equation (2) and (3), (4), or (5).

Unfortunately, GARCH under empirical measure does not price either options (Barone-Adesi, Engle & Mancini, 2008) or VIX accurately. As a matter of fact, empirical GARCH under-estimates VIX consistently (Hao & Zhang, 2010). The difference between the observed market VIX and GARCH-estimated VIX is termed the variance risk premium by Hao and Zhang (2010). The underestimation is also confirmed by the empirical investigation of this paper.

Risk-neutral GJR GARCH parameters calibrated by the market prices of traded options do a much better job in pricing options (Barone-Adesi, Engle & Mancini, 2008; Liu & Xiang, 2012). Further, Barone-Adesi et al. (2008) mentioned briefly that the Risk-neutral GJR GARCH parameters can be used to simulate daily variances in order to price VIX. Their idea was adopted by a later paper (Byun & Min, 2012).

Unfortunately, it is quite expensive computationally to obtain the risk-neutral GARCH parameters by simulating price paths, pricing the options, and minimizing pricing errors (Barone-Adesi, Engle & Mancini, 2008; Byun & Min, 2012). Since VIX is by now well-established, this paper proposes to calibrate the GARCH parameters by VIX directly via the closed-form Equation of (2). The saving of doing
this is twofold. First, prices of traded options are not needed. Second, price paths and daily variances do not have to be simulated. Furthermore, because market VIX is computed from prices of traded options, the new scheme obtains the risk-neutral measure directly through Equation (2), without having to use the risk-free interest rate. This makes the new approach simpler additionally.

With the VIX-calibrated risk-neutral GARCH parameters, VIX can be priced directly by Equation (2). Once again, daily variances do not have to be simulated.

4. EMPIRICAL STUDY

4.1. Data Description

Since CBOE does not provide historical data for the S&P 500 Index, both daily VIX and the S&P 500 Index from Yahoo!Finance are used in this study. A comparison of VIX from CBOE and Yahoo shows only very minor differences for a few days. It seems that the quality of Yahoo!Finance data should not be a problem for our purpose.

On 22 September 2003, CBOE modified the method for computing VIX. Accordingly, this paper divides the time series of VIX into two sub-periods. Phase I is between 2 January 1996\(^2\) and 19 September 2003; Phase II is from 22 September 2003 to 31 January 2012. Table I provides a summary description of the VIX data. Apparently, the long-term average of VIX is somewhat stable, but VIX shows more pronounced variation in Phase II.

Table I here

\(^2\) This choice makes the two parts have roughly the same trading days.
4.2. Computational Details and Results Analyses

Assume $t$ is the valuation date, and $t-1$ is the calibration date. Following Barone-Adesi et al. (2008), the paper utilizes 3500 daily returns of the S&P 500 Index between $t-3500$ and $t-1$. $\sum_{i=1}^{3500} \left( \ln v_{t-i} + \epsilon_{t-i}^2 / v_{t-i} \right)$, the negative of the maximum-likelihood function (Hull, 2009), is minimized via the Nelder-Mead algorithm (Press, Teukolsky, Vetterling & Flannery, 2002) to obtain the optimal set of empirical GARCH parameters. The reported average parameters for the two phases are quite close (Table II).

Table II here

Denote the market VIX level for the calibration date by $VIX_{t-1}^m$. The objective function $\{\sqrt{84000[30V_L^i + (v_t - V_L^i)(1-\xi^{30})/(1-\xi)] - VIX_{t-1}^m}\}^2$, where $v_t$ (and $\epsilon_t$ as well) is computed via Equation (1) using the empirical GARCH parameters, is again minimized via the Nelder-Mead algorithm\(^3\) to obtain the risk-neutral GARCH parameters (Table III). The risk-neutral parameters are markedly different from their corresponding empirical ones. Further, the parameter $\xi$ for Phase I for all three GARCH models is larger than one, which means that the risk-neutral models are mean-fleeing (Hull, 2009).

Table III here

\(^3\) Nelder-Mead is sensitive to the initial choice of the variables’ characteristic lengths, which are obtained through extensive testing of the three GARCH models and different for estimating the empirical GARCH and calibrating.
With the empirical and risk-neutral parameters, the VIX level for the valuation date can then be computed via Equation (2), where $v_{t+1}$ is once again computed via Equation (1) using the empirical GARCH parameters. The results are summarized in Table IV.

Table IV reports three error measures. The mean of pricing errors (MPE) is defined as $\sum_{j=1}^{N} (VIX^c_j/VIX^m_j - 1)/N$, where $VIX^c_j$ is the computed VIX and $VIX^m_j$ the market VIX for date $j$. The mean of absolute pricing errors (MAE) is obtained via $\sum_{j=1}^{N} |VIX^c_j/VIX^m_j - 1|/N$. The root mean of square pricing errors (RMSE) is computed by $[\sum_{j=1}^{N} (VIX^c_j/VIX^m_j)^2]/N]^{0.5}$.

Both MPE and MAE errors under the empirical measure are rather large for all three GARCH models. MPEs are negative but close to MAEs in absolute value, which implies that on average the empirical GARCH underestimates VIX. This underestimation of VIX by empirical GARCH can be regarded as the variance risk premium that is not present in the observed prices of the underlying S&P 500 Index. Interestingly, the mispricing for Phase I is around 25%, but only about 14% for Phase II. This seems to suggest that the variance risk premium becomes smaller in Phase II.

Under the calibrated risk-neutral measure, all errors are quite small and roughly the same for Phases I and II. MPEs are really small while MAEs are one order of magnitude larger but still less than 5%. This implies that on average the errors from under-pricing and overpricing cancel out. Remarkably, all three GARCH models price
VIX accurately.

Among the three GARCH models, G11 has the smallest errors for pricing VIX under the empirical GARCH measure, but displays the biggest pricing errors under the risk-neutral measure. The latter could be understandable since G11 uses only three parameters to fit the data, but the former seems hard to explain. Finally for HN, the situation of pricing errors under the empirical and risk-neutral measure is reversed against G11.

5. CONCLUSIONS

This paper proposes a simple GARCH approach to pricing VIX. With closed-form formulas for computing VIX based on symmetric GARCH(1,1), asymmetric GJR GARCH(1,1), and asymmetric Heston-Nandi GARCH(1,1) models, the GARCH parameters in these formulas can be calibrated by the market VIX of the previous trading day. The calibrated parameters are risk-neutral, and can be used with these formulas to price (or forecast) VIX.

With empirical GARCH parameters estimated from 3500 daily returns for the S&P 500 Index, the formulas under-price VIX by 19.9~33.9% for the period of 2 January 1996 - 19 September 2003 and by 12.0~14.4% for the period of 22 September 2003 - 31 January 2012. This underestimation could be referred to as variance risk premium.

Using risk-neutral GARCH parameters calibrated from the market VIX, these formulas reduce dramatically the pricing errors to 0.1~0.2%. Importantly, the
differences among the three GARCH models and between the two sub-periods are almost negligible.

In summary, the proposed GARCH method can price VIX accurately. Further, with closed-from analytic formulas, the new approach is also computationally efficient. Finally, those results could in principle be extended and applied to the pricing of VIX futures, VIX options, and even S&P 500 Index options.
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Table I. Description of the VIX prices.

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std Dev</th>
<th>No. Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase I</td>
<td>12.00</td>
<td>45.74</td>
<td>23.58</td>
<td>5.73</td>
<td>1942</td>
</tr>
<tr>
<td>Phase II</td>
<td>9.89</td>
<td>80.86</td>
<td>21.08</td>
<td>10.51</td>
<td>2106</td>
</tr>
</tbody>
</table>

Table II. Empirical GARCH parameters.

<table>
<thead>
<tr>
<th></th>
<th>$\omega \times 10^6$</th>
<th>$\alpha \times 10^2$</th>
<th>$\beta$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G11</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>Mean</td>
<td>1.543</td>
<td>8.175</td>
<td>0.9056</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.628</td>
<td>1.414</td>
<td>0.0188</td>
</tr>
<tr>
<td>Phase II</td>
<td>Mean</td>
<td>1.193</td>
<td>8.279</td>
<td>0.9104</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.396</td>
<td>0.719</td>
<td>0.0087</td>
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<td><strong>GJR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>Mean</td>
<td>2.208</td>
<td>5.437</td>
<td>0.8933</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.509</td>
<td>0.478</td>
<td>0.0113</td>
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<td>Phase II</td>
<td>Mean</td>
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<td>5.495</td>
<td>0.8954</td>
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<td></td>
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<td>0.194</td>
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<tr>
<td><strong>HN</strong></td>
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<tr>
<td>Phase I</td>
<td>Mean</td>
<td>2.237</td>
<td>2.134</td>
<td>0.8223</td>
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<tr>
<td></td>
<td>Std Dev</td>
<td>0.986</td>
<td>0.683</td>
<td>0.0450</td>
</tr>
<tr>
<td>Phase II</td>
<td>Mean</td>
<td>1.710</td>
<td>2.127</td>
<td>0.8061</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>1.225</td>
<td>0.539</td>
<td>0.0650</td>
</tr>
</tbody>
</table>

Phase I: 2 January 1996 - 19 September 2003. Phase II: 22 September 2003 - 31 January 2012. The GARCH models are $\ln(S_t/S_{t-1}) = \mu + \epsilon_i, \epsilon_t = \omega + \beta \epsilon_{t-1} + u_{t-1}$, where $\epsilon_t = \sigma_t^2, \epsilon_t = \sigma_I z_t$, and $z_t$ is the empirical random variable with a mean of zero and a variance of one. For G11, $u_t = \alpha \epsilon_t^2, \xi = \alpha + \beta$; for GJR, $u_t = (\alpha + \beta I\{z_t < 0\}) \epsilon_t^2$, where $I\{z_t < 0\}$ is the indicator function, $\xi = \alpha + \beta + 0.5 \gamma$; for HN, $u_t = \alpha (z_t - \gamma \sigma_t)^2, \xi = \alpha \gamma^2 + \beta$.

Parameters are estimated by minimizing $\sum_{i=1}^{3500} (\ln \epsilon_t^2 + \epsilon_{t-1}^2 / \sigma_t^2)$ with 3500 daily returns.
Table III. Risk-neutral GARCH parameters.

<table>
<thead>
<tr>
<th></th>
<th>ω×10^6</th>
<th>α×10^2</th>
<th>β</th>
<th>ξ</th>
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<tr>
<td><strong>G11</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>Mean 3.875</td>
<td>5.273</td>
<td>0.9532</td>
<td>1.0059</td>
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<tr>
<td></td>
<td>Std Dev 0.461</td>
<td>0.351</td>
<td>0.0198</td>
<td>0.0217</td>
</tr>
<tr>
<td>Phase II</td>
<td>Mean 4.026</td>
<td>4.939</td>
<td>0.9305</td>
<td>0.9798</td>
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<tr>
<td></td>
<td>Std Dev 0.650</td>
<td>0.566</td>
<td>0.0312</td>
<td>0.0355</td>
</tr>
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<td><strong>GJR</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Phase I</td>
<td>Mean 4.042</td>
<td>5.593</td>
<td>0.9218</td>
<td>5.986</td>
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<tr>
<td></td>
<td>Std Dev 0.260</td>
<td>0.375</td>
<td>0.0195</td>
<td>0.047</td>
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<tr>
<td>Phase II</td>
<td>Mean 4.093</td>
<td>5.320</td>
<td>0.8988</td>
<td>6.040</td>
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<tr>
<td></td>
<td>Std Dev 0.291</td>
<td>0.464</td>
<td>0.0284</td>
<td>0.096</td>
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<tr>
<td><strong>HN</strong></td>
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<tr>
<td>Phase I</td>
<td>Mean 3.248</td>
<td>2.455</td>
<td>0.8759</td>
<td>233.7</td>
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<td></td>
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<td>0.192</td>
<td>0.0254</td>
<td>4.5</td>
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<tr>
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<td>Mean 3.136</td>
<td>2.357</td>
<td>0.8463</td>
<td>231.8</td>
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<td></td>
<td>Std Dev 0.297</td>
<td>0.191</td>
<td>0.0412</td>
<td>4.3</td>
</tr>
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\[ \sqrt{84000[30V_{t-1} + (V_t - V_{t-1})(1 - \xi^3)/(1 - \xi)] - \text{VIX}_V^m} \]

where \( \text{VIX}_V^m \) is the market VIX level for the calibration date, \( \xi = \alpha + \beta, V_t = \omega/(1 - \xi) \) for G11, \( \xi = \alpha + \beta + 0.5\gamma, V_t = \omega/(1 - \xi) \) for GJR, and \( \xi = \alpha\gamma^2 + \beta, V_t = (\omega + \alpha)/(1 - \xi) \) for HN. Further, \( v_t \) is computed by using the empirical GARCH parameters

via \( \ln(S_t/S_{t-1}) = \mu + e_t, v_t = \omega + \beta_v v_{t-1} + u_{t-1}, \)

where \( v_t = \sigma_t^2, e_t = \sigma_t z_t, u_t = \alpha e_t^2 \) for G11,

\[ u_t = (\alpha + \gamma I[z_t < 0])e_t^2 \]

(where \( I[z_t < 0] \) is the indicator function) for GJR, and \( u_t = \alpha(z_t - \gamma\sigma_t)^2 \) for HN.
Table IV. Errors of VIX pricing by GARCH.

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
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<tr>
<td></td>
<td>MPE (%)</td>
<td>MAE (%)</td>
<td>RMSE</td>
<td>MPE (%)</td>
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<tr>
<td>Phase I</td>
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<tr>
<td>G11</td>
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<td>21.5</td>
<td>5.99</td>
<td>0.2</td>
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<tr>
<td>GJR</td>
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<td>24.6</td>
<td>6.70</td>
<td>0.2</td>
</tr>
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<td>HN</td>
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<td>34.0</td>
<td>9.50</td>
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<td>19.2</td>
<td>8.16</td>
<td>0.1</td>
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</tbody>
</table>

Phase I: 2 January 1996 - 19 September 2003. Phase II: 22 September 2003 - 31 January 2012. For both the empirical and risk-neutral pricing, VIX is computed via $\sqrt{84000[30V_t^2 + (v_{t+1} - V_t^2)(1 - \xi^{30})/(1 - \xi)]}$, where

$$\xi = \alpha + \beta, V_t^2 = \omega/(1 - \xi)$$ for G11, $$\xi = \alpha + \beta + 0.5\gamma, V_t^2 = \omega/(1 - \xi)$$ for GJR, and 

$$\xi = \alpha\gamma^2 + \beta, V_t^2 = (\omega + \alpha)/(1 - \xi)$$ for HN. Further, $v_{t+1}$ is obtained by using the empirical GARCH parameters via $\ln(S_t/S_{t-1}) = \mu + \epsilon, v_t = \omega + \beta v_{t-1} + u_{t-1},$ where $v_t = \sigma_t^2$, $\epsilon = \sigma_t z_t$, $u_t = \alpha \epsilon_t^2$ for G11,

$u_t = (\alpha + \gamma I\{z_t < 0\})\epsilon_t^2$ (where $I\{z_t < 0\}$ is the indicator function) for GJR, and $u_t = \alpha (z_t - \gamma \sigma_t)^2$ for HN. MPE: mean of pricing errors. MAE: mean of absolute pricing errors. RMSE: root mean of square pricing errors.